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Physics: A Human Endeavour

Unit I MOTION

Text

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"... I believe, basically, we have not been cautious enough of the meaning of science in our generation, to teach it in a way which would be understood and appreciated and felt by the students. We have very little of the positive values of science outside of the applications which are obvious to anybody living in this age. In other words, my claim is, and this is something we should discuss, that we have not been teaching our science in a humanistic way. We have been teaching science at every level, in a certain sense, as a certain bag of tricks which the bright boy or girl could learn and show off with, or at least get a great deal of pleasure out of—the same kind of pleasure, but not quite as sharp, as he would get out of plane geometry.

Now, science is a very different thing . . . it is an adventure of the whole human race to learn to live in and perhaps to love the universe in which they are. To be a part of it is to understand, to understand oneself, to begin to feel that there is a capacity within man far beyond what he felt he had, of an infinite extension of human possibilities—not just on the material side (whatever that may mean, because the more we study the material side, the more and more it recedes) . . . .

So what I propose as a suggestion for you is that science be taught at whatever level, from the lowest to the highest, in the humanistic way. By which I mean, it should be taught with a certain historical understanding, with a certain philosophical understanding, with a social understanding and a human understanding in the sense of the biography, the nature of the people who made this construction, the triumphs, the trials, the tribulations."

From the address of I. I. Rabi at AAAS meeting of Educational Policies Commission, 27 December 1966, Washington, D.C.

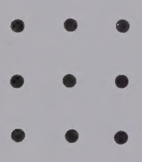
Preface

Physics: A Human Endeavour is an examination of some parts of man's attempt to understand the physical world. We call physics a "human endeavour" because the development of our understanding of the physical environment is the result of interactions between human minds and nature.

It is perhaps most important in the world of today, a world in which there is an "information explosion", to seek a broad understanding of the ways in which information is obtained, and the limitations, applications and implications of such knowledge.

In order to achieve this understanding, we shall not only examine the physics of the twentieth century, but we shall study the evolution of scientific thought from the past to our contemporary point of view.

This examination can reveal to us one characteristic of the human mind that is particularly interesting. It is symbolized by the nine dots we have used as the basis for the simple puzzle below.



Draw the nine dots on a piece of paper.
Join all nine dots by
four straight lines without
lifting your pencil from the page.

Try it!

If you have not succeeded yet, the answer is found on the back cover.

It is fun to watch a group of people attempt this problem. Invariably, most people draw lines in all directions, but stop when they reach the outside row of dots. They seem hemmed in by those outside dots as if they feel them representing some kind of boundary on the problem. In order to succeed, your pencil must break through this self-imposed boundary (there was nothing in the instructions to suggest such a boundary existed!) You may know other puzzles of this kind, or remember problems that you have solved only after breaking through some boundary on your thinking. We shall call such mental boundaries paradigms. Most people, faced with a problem, search for some familiar pattern or model on which to base their thinking. That is, consciously or unconsciously, they seek to establish a paradigm. We suggest that by examining some of the major paradigms that have existed in the past, and the ways in which they were broken, we can go a long way towards recognizing the paradigms of today, an important ability for success in the discipline of physics, or any other human endeavour. As the distinguished Canadian critic, Northrop Frye, has said, "It is a part of the whole educational process to recognize as far as possible the extent of one's own conditioning and to try to go as far as humanly possible in becoming aware of what one's own assumptions and axioms are." (Northrop Frye in an interview with Bruce Mickleburgh in "Monday Morning", Sept./72.)

Now let us make some specific comments on the structure of the course.

There are six units in all. Each has one or two central themes, and makes its own kind of contribution to your knowledge of physics.

Much of what happens in the universe involves *motion*, and it is important to be able to describe how and why objects move. These areas, called kinematics and dynamics, form the basis of the information in Unit I.

Unit II is a further discussion of the laws of motion. We expand upon the concepts of simple motion by taking the laws out the limited sphere of the earth's gravity and applying them to the motions of our solar system and ultimately, the universe. As you will see, the analogy to the dot problem applies. Man had to expand beyond the earth (the limits of the dots) into the universe (beyond the dots) to continue his search for knowledge. Where is man located in the vastness of space? Where are the stars? Is there an order to the Universe? The variety of answers to these questions, and the knowledge that the answers are still changing, is the concern of Unit II.

In the third unit, the formulation of a few of the famous "Conservation Laws" is discussed with emphasis on the fascinating properties of momentum, energy, and energy transfer by waves. The development of man's sources of useful energy and the possibility of a 20th century energy crisis are also considered.

Unit IV concentrates on the use of "models" in science, particularly focussing on the classic conflict between two rather successful models to describe the nature of light—the particle model proposed by Sir Isaac Newton, and the wave model championed by the Dutch physicist Christian Huygens. Although we shall see how this particular conflict was resolved, we shall also see how a similar problem about the nature of light still exists today.

The story of electricity, the relationship between electricity and magnetism, and the tremendous impact that developments in this field of physics have had on our way of life is traced in the fifth unit.

Finally, the world of the very small, the world of objects of atomic and nuclear sizes, the world of X-rays, gamma rays and radioactive particles, the world of nuclear fission, fusion, and atomic energy, is probed in Unit VI.

Throughout the text, you will find questions at the end of most sections. These are usually straightforward questions which can be answered directly from the preceding material. They focus on the major points made in the section. Problems, graded into A and B levels of difficulty, are provided at the end of each chapter to help you develop further your ability and insight into the issues in the text. You will find as you work your way through the book that many of the End-of-section and End-of-chapter questions are marked with asterisks. This indicates that the problem is a discussion type and that the answer will not be found at the end of the book in the answer section.

We believe that no student taking this course should limit his reading to this text alone. One of the habits a student must acquire is to pursue a given topic from several points of view. To this end, there are many references made throughout the text to books that have been read and enjoyed by other students studying physics at your level.

Finally, we should like to point out to you the importance of laboratory work. Ultimately, all science is guided by the phrase "theory guides, but experiment decides". Measuring properties of nature directly usually raises a host of difficulties that are not met when working through the same material with pencil and pad. Some whimsical scientists (original sources unknown by the authors), have given concise expression to feelings that any student of science can share from time to time in the forms of "*Murphy's Law*"—If anything can go wrong—it will and "*Allen's Axiom*"—When all else fails, read the instructions.

We hope that you don't find these applicable to yourself too often this year, but when you do, smile, re-organise, and try again!

The authors gratefully acknowledge their colleagues who have taken the time to read and criticise their manuscript. In addition, the authors appreciate, with thanks, the co-operation received from their publisher, Holt, Rinehart and Winston of Canada Limited.

The authors and publisher of *Physics: A Human Endeavour* wish to acknowledge their indebtedness to the *Harvard Project Physics Text* for kindly permitting them to follow the format and adapt the content of the *Project Physics Course.*

Prologue

What Is Physics All About?

Physics is what those scientists who are called *physicists* do, when they are doing *physics.*

—Author unknown

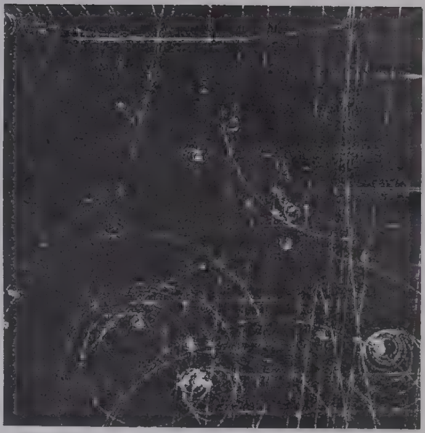
A peak in the Andes mountains of Chile might at first seem a strange place to find a young Canadian trying to answer one of the oldest questions in the development of science. The mountain at Las Campanas is the location of a 60 cm reflecting telescope operated by the University of Toronto. The young man, Bill Harris, is a graduate student at the university who has spent many evenings using this telescope to photograph some groups of stars called globular clusters. An answer to the question: How old is the universe? is what Bill hopes his research will achieve.

Globular Star Cluster in Hercules.

In Room 343 at the MacLennan Laboratory, on the University of Toronto campus, a young lady surrounded by books, research papers and boxes of computer cards is attempting to solve a problem concerning the fundamental particles that make up our universe. Peggy Donnelly, a graduate student in theoretical physics, is hoping to use the computer and mathematical analysis to better understand the interaction between a π - meson and a deuteron. From this work she may make a contribution to our understanding of these elementary particles.

These two students share in their search for a better understanding of our physical universe. They differ in the manner in which they approach their problem. Bill Harris is an experimenter. He has spent the past year at several observatories taking photographs which he will analyse to determine the brightness of globular clusters in our galaxy. He hopes to combine his observations with those of another experimenter who is photographing clusters in distant galaxies. From their analysis they may be able to calculate a value for the age of the universe.

Peggy Donnelly is a theoretician. She views the interactions between elementary particles as mathematical equations, or as she calls them, approximations, which must be analysed and improved. She will try to approximate the interaction between the π - meson and the deuteron by equations and then use the computer to test the effect of the assumptions used in her equations. The final test for her work could be the agreement of her predictions with the results of the actual experiment between the π - meson and the deuteron. However, Peggy does not confine herself to her desk and computer. She has travelled to institutes in Yugoslavia and Sicily to meet with other theoretical physicists to learn new ideas and to share problems.



Liquid hydrogen bubble chamber photograph taken at Brookhaven National Laboratory, showing interaction between elementary particles.

The questions asked by a physicist in the 20th century range from those of the astrophysicist involving the birth, death and age of a star, or the universe itself, to those of the theoretical nuclear physicist about the building blocks of matter. These questions are concerned with the many forms of energy about us, and the interaction between energy and matter. The physicist studies many such interactions. Biophysicists are concerned with interactions between radiation and human tissue. Geophysicists try to analyse the effects of heat and magnetism on the rocks of the earth's crust. Others are searching for new sources and methods of releasing energy to satisfy the demands of an energy hungry world.

But physics is not only about the interaction between energy and matter. It is also a story of interactions between a most interesting cast of characters, called physicists, and the society in which they lived. The decision to build a Canadian telescope in Chile involved politicians as well as scientists. Nuclear physicists are concerned not only with their search for scientific knowledge, but the manner in which this knowledge is used by man.

Our initial answer to the question: What is physics all about? cannot be complete. Let us say for now that physics involves people called physicists who ask and try to answer questions about interactions between energy and matter while they interact with the society in which they live. Perhaps after reading about the development of physics over the past 2000 years and after carrying out many experiments yourself you will come to your own understanding of what physics is all about. Perhaps too, when we reach the epilogue to Unit VI we will be able to let you know the degree of success Peggy Donnelly and Bill Harris have had in their attempts to better understand our physical universe.

Chapter 1 *Physics Begins*

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Study for "Dynamism of a Cyclist" (1913) by Umberto Boccioni. Courtesy Yale University Art Gallery.

Physics Begins

Chapter One

Introduction

Consider a few fundamental questions concerning the physical world.

1. Of what is matter composed?
2. What causes an object to fall to the ground?
3. Can a perfect vacuum exist?

The answers that you give to these questions will depend upon the amount of scientific reading you have done before taking this course. You may have used such words as elements, atoms, protons, or electrons in answering question 1; gravity or weight, in question 2. And you possibly answered in the affirmative to question 3, thinking of vacuum pumps, or "space ships". But no matter what your answers were, we suspect that they all fell within a certain framework of thinking, an attitude of mind that is characteristic of people living in our present era.

This attitude, and the words and concepts that we presently accept as being adequate to describe physical reality, are a relatively recent development. For over 2000 years (from 600 B.C. to about 1600 A.D.) the world was described from a very different perspective. We suggest that you can best appreciate our present way of thinking by considering first the views of the ancient Greek philosopher-scientists.

1.1 The Perspective of the Greeks

In any attempt to summarize an entire system of thinking in a few paragraphs there are bound to be oversimplifications. However, we can develop a feeling for the spirit of the times by looking at the description of the world put forward by the Greek civilization in the period 600 B.C. to 200 A.D. We shall pay particular attention to the works of Aristotle, as his teachings in many fields later became the most influential of all the works of the Greeks.

Aristotle, who was born in 384 B.C., was a son of the physician to Philip, King of Macedonia, and was himself the tutor of Alexander the Great. He was the greatest collector and systematizer of knowledge that the ancient world produced. He was a tireless writer, and although the bulk of his work is lost, enough remains to stagger the imagination. Substantial works on philosophy (logic and metaphysics), meteorology, natural history, rhetoric, politics, astronomy, biology, and physics are credited to him. Some scholars doubt that it was all the work of one man.

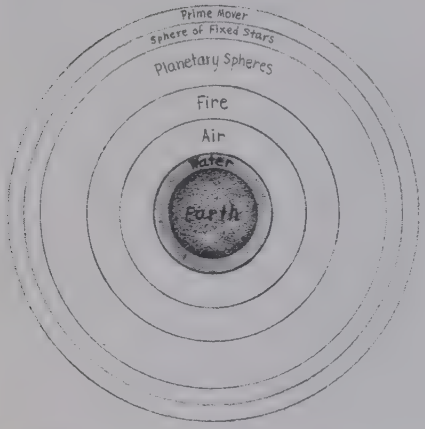


Diagram of the medieval concept of the world structure.

Aristotle believed that all things on or near the earth were formed from four fundamental "elements", Earth, Air, Fire, and Water. These elements were not thought of as identical with the natural materials for which they were named. Ordinary water, for example, was thought of as a mixture of all four elements, but it was mostly the element Water.

Each of the four elements was thought to have a "natural place". Earth itself occupied the central, lowest, and most fundamental place. In fact, it was the centre of the entire universe. Water was next, then Air. Fire, which seemed to have the ability to rise through Air, was accorded the highest place. Objects would move to their most natural place, depending upon the particular mixture of elements in the object. Bodies that were composed mostly of the element Earth would fall to the earth, even if they had to pass through Air and Water to get there. When water boiled, the element Water would be joined by the element Fire, whose higher natural place would cause the mixture to rise as steam.

From *quinta essentia* meaning fifth essence. In earlier Greek writings the term used was *aether.*

According to Aristotle the stars, planets, and other celestial bodies differed fundamentally in composition and behaviour from objects on or near the earth. The celestial bodies were believed to contain none of the four ordinary elements, but instead to consist solely of a fifth element, the quintessence. The heavenly bodies were regarded as being in their natural place (in heaven), not striving to get nearer or farther from the earth, and therefore they moved in perfect circles around the centre of the universe (earth).The motions of celestial objects then, were regarded as following a completely separate set of laws from those of terrestrial bodies.

According to Aristotle, all motions in the physical world occurred in order to preserve the general order of the universe. It was more important that the explanation of a particular event should fit into the philosophical-theological pattern of the time than it was to be strictly "scientifically" true. Philosophic truth took precedence over scientific truth.

Answers to end-of-section questions that are marked with asterisks do not have answers at the back of the book.

* **Q1** Aristotle's theory of motion of the elements toward their natural place seems to be supported to a great extent by experience. Water bubbles up through the earth at springs. Fire seems to rise up and disappear as if reaching for the sky. Can you think of other examples?

1.2 Two Types of Motion on the Earth

Motion on the earth was considered by Aristotelians in two separate categories.

Natural Motion

Natural motion was exhibited by any body moving freely to its "natural place". A rock fell to the ground not because of an attraction by the earth, but merely because it had a natural tendency to fall there. (After all, it is made mostly of the element Earth!) Aristotle would see no need to explain the cause of this motion any further than this. This is an excellent illustration of a fundamental principle of Aristotelian physics; *it is not the relation of a body to other bodies which determines its behaviour, but qualities that inhere in the body itself.*

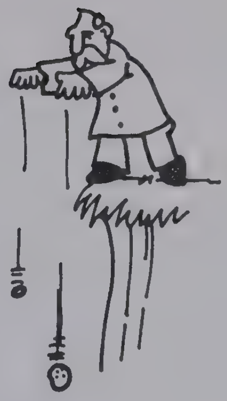
This explanation of natural motion has an important implication. Since a heavy body would have more Earth element than a smaller one, it would be expected to hit the ground first if both were released from the same height. You might admit from your experience that this is often true. A golf ball will hit the ground before a ping-pong ball, if they are dropped from a great enough height. In addition, as we watch them fall, they *seem* to fall at constant speeds (the heavier ball falling faster). Objects fall more slowly through water than through air, and hence the resistance of the medium must be a factor controlling motion. Aristotle decided then, that bodies falling to the earth:

1. fell at constant speed;
2. fell faster the greater their weight;
3. fell slower the greater the resistance of the medium.

You might object that the only reason a light object falls slower than a heavy one is the effect of air resistance. Take away the air, you may argue, and you will see quite different behaviour. Aristotelians, however, would not be at all impressed by this line of reasoning. They certainly could not actually remove the air from the path of falling objects (The "vacuum pump" was not invented until 1654). But, more than this, it was fundamental to the Aristotelian system that a vacuum was, *in principle*, impossible to create. "Nature abhors a vacuum" was the spirit of their thinking. Thus, to invite the human mind to contemplate what would happen to objects if they were allowed to fall under conditions that (according to the Greek mentality) could not exist, would be useless speculation, not at all connected with the real world that is the concern of physics.

It was eventually seen that a falling body does not move with uniform speed, but speeds up (accelerates) as it falls. Some Aristotelians thought that this was associated with its impending arrival at its natural place, the earth. In other words, a falling object was thought to be like a tired horse that starts to gallop as it approaches the barn. Others claimed that when an object falls, the weight of air above it increases, while the column of air below it decreases, thus offering less resistance to its fall.

When a falling object finally reaches the ground, as close to the centre of the earth as it can get, it stops. There, in its natural



According to Aristotelian physics, bodies fall to their natural place. Heavier bodies fall faster than lighter ones.

Bodies falling to the ground move quite rapidly. Without modern instruments it is difficult to be sure of the accuracy of any statement made about what happens to their speed as they fall. To slow down their motion so that falling objects could be studied more easily, Aristotle used water instead of air as the medium. In water, objects fall much more slowly than in air, with heavier objects falling faster than lighter ones.

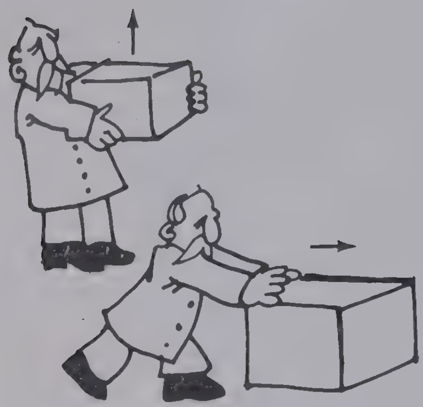
See Experiment 1.2, pg. 104.

Another argument against the possibility of a vacuum could be deduced from Aristotle's theory: if the rate of fall is equal to the weight divided by the resistance and the resistance of a vacuum is zero, then the rate of fall of *all* bodies must be infinite in a vacuum. But that is absurd. Hence a vacuum is impossible.

place, it remains. Rest, being regarded as the natural state of objects on earth, required no further explanation.

Violent Motion

Not all motions, however, involve bodies in a natural kind of motion. An object pushed across the ground is obviously not moving to its natural place, as it is already in its natural position, on earth. To sustain "unnatural" motion, Aristotle argued, the object must be forced by some outside object. If the source of the push or pull is removed, the object will stop. This kind of forced motion he called *violent motion.* Lifting an object up in the air would be causing it to undergo a violent motion, and again, a force is required to sustain the movement.



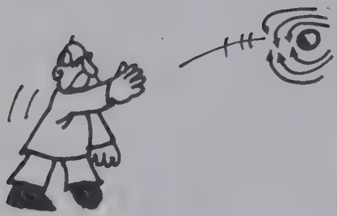
According to Aristotle, these were violent motions. Forces were needed to sustain the motion of the objects.

As another example of violent motion, consider Aristotle's explanation of why a stone thrown from the hand keeps moving after the force of the thrower's hand is removed. Aristotle taught that when the rock is being pushed forward by the thrower's hand, it sets a layer of air in motion. Just as the moving rock leaves the hand, air rushes in behind the rock to fill up the space left when the rock moves (remember, nature abhors a vacuum), and as the air rushes in behind the rock, it pushes the rock forward. Eventually, however, the rock's natural tendency to fall becomes more important than its violent motion, and it falls to the ground.

Q2 Explain what Aristotle meant by natural and violent motion. Give an example of each type.

Q3 According to Aristotle, which would hit the ground first if released from the same height, a sphere with 10 units of weight, or one with 5 units of weight? Why?

Q4 Answer the three questions posed at the beginning of the chapter as an Aristotelian teacher might have.



This diagram shows why a stone keeps moving when thrown, according to Aristotle.

1.3 The Status of Aristotle's Teachings by the 16th Century

We have devoted but a few pages here looking at only a very small portion of the ideas of one of the Greeks who contributed to early science. Reference to works by others of the age, such as: Democritus, Hippocrates, Archimedes, Euclid, and Ptolemy produces a deep respect for the achievements of ancient Greek culture. This was indeed one of the great periods of intellectual advancement in history.

The following quotation effectively illustrates the different ways in which the ancient Greeks and Romans would probably solve the same problem. "If someone needed to know the distance between Rome and Paris, the Greeks would use star positions and geometry to calculate the distance, while the Romans would send an army to pace it out."

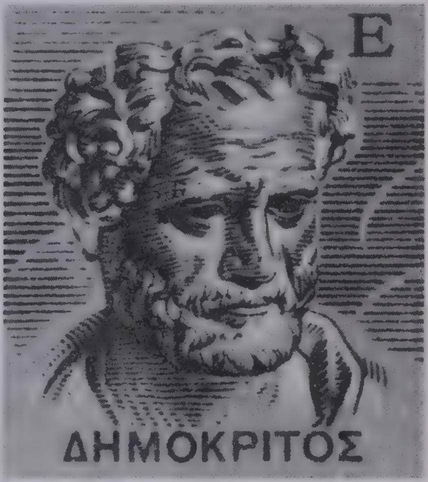
In 146 B.C., the Greeks were conquered by a new people, the Romans. With all their genius as soldiers, engineers, and administrators, the Romans had little interest in natural philosophy (science), and the cultivation of science in the Western World was neglected.

During the first few centuries of the Christian era, the civilized world was plagued by social, political, and religious instability. As Christian theology developed, fierce quarrels broke out concerning the proper interpretation of Greek philosophy and the manner in which it should be reconciled with theology.

Aristotle's notion that terrestrial laws were not valid for the celestial regions, and his belief in a geocentric (earth-centred) universe fit in well with the theological views of the time. Wherever conflict arose, however, the issue was settled by simply reinterpreting the works of the ancients. The idea that perhaps those works might contain errors was not seriously considered. Any discussion of science was centred on the theology that was so closely associated with it, rather than on science alone. Natural science, in fact, came to be considered pagan. In 391, the Bishop of Alexandria ordered much of the great library at Alexandria destroyed, and in 529, Justinian closed the Academy at Athens. Tens of thousands of manuscripts were lost or destroyed. The prevailing atmosphere was not at all favourable to progress in science.

The period of history from about 300 to 1000 A.D. is often referred to as the "Dark Ages". There were relatively few contributions made to learning of any kind during that period.

During the crusades of the thirteenth century, many of the works of Aristotle and other Greeks were rediscovered, and translated into Latin by monks and clerics. Around 1250, one famous monk, Thomas Aquinas, succeeded in reconciling Aristotelian science, with Christian theology. In his great treatise, *Summa Theologica*, Aquinas established Aristotle as the major authority in scientific and philosophical matters. The thought of the early Middle Ages came to be entirely dominated by Aristotelian physics which had become a set of dogmatic assumptions elaborated in detail by scholars and imposed with the full authority of the church.



This picture of a statue of Democritus is found on a Greek Stamp issued to Commemorate the opening of a nuclear research institute.

* **Q5** The Aristotelian explanation of motion should not be dismissed lightly. Great intellectuals of the Renaissance period such as Leonardo da Vinci, who, among other things, designed artillery for launching projectiles, apparently did not challenge the Aristotelian explanation. One reason for the longevity of Aristotle's ideas is that they are so closely aligned with our common-sense ideas. In what ways do your common-sense notions of motion agree with Aristotle's?

1.4 Resistance to Change

Several influences stood in the way of radical changes in Aristotle's theory of motion. Aristotle believed that mathematics was of little value in describing physical phenomena, and he put great emphasis upon direct, qualitative observation as the basis for theorizing. His theories then, agreed with much of human experience. Simple qualitative observation was very successful in Aristotle's biological studies, but real progress in physics began only when the value of mathematical prediction and detailed measurement was recognized. We must also remember that Aristotle's major works were in the field of politics and philosophy. His thinking in these areas was, and still is, regarded with the highest respect. Even today, men who achieve greatness in one area of endeavour often influence areas outside their specialty. Finally, Aristotle's theory presupposed that all motion occurred in order to achieve some purpose, to satisfy some preordained goal. This view was naturally acceptable to the medieval church

Suggested for Further Reading.

W.E.K. Middleton, *The Scientific Revolution.* Toronto: CBC Publications, 1963.

This short, inexpensive paperback provides excellent background material for many of the ideas discussed in units I and II.

and was one of the reasons why Aristotelian physics was adopted with such a sense of certainty by the Church.

A number of scholars in the fifteenth and sixteenth centuries had a part in the evolution of a new way of "doing" science. One of these, Galileo Galilei, has been called "the father of modern science". Galileo showed how to study simple motions under controlled conditions and to describe their behaviour mathematically. This work not only paved the way for other men to describe and explain the motions of everything from pebbles to planets; it also began an intellectual revolution which led to what we now consider modern science. We shall examine his work in Chapter 3.

*Q6 By using reason alone, not by referring to any knowledge of "modern physics" that you may have, can you offer any criticisms of Greek science?

The Early Scientists

Sometime around 600 B.C., emerging from a past in which every natural object or event was thought to be inhabited by or caused by some supernatural being, there developed among the peoples living in the Mediterranean region which we now call Greece, a new way of viewing the world. Analysed in its simplest terms, we may say that there began to be offered explanations of natural events without involving the aid of any power outside nature herself. The beginnings of a "scientific" attitude appeared.

It is convenient to identify three main periods and centres of activity in Greek science.

The Early Ionian Philosophers

Thales (600 B.C.)

The most important early Ionian scientist was Thales (630 to 540 B.C.). His interest in science originated with travels to Egypt and Babylonia where he developed a knowledge of *practical* geometry. He devised a method of determining the distance of ships at sea, based on the doctrine of similar triangles, and presented an improvement on methods of navigating by the stars. Thales' significant contribution was, however, his development of rudiments of a theory of geometry. He observed that geometric figures had certain properties (such as: the sum of the measures of the angles of a triangle equals 180°) and asked, Could this surprising fact be deduced from something more obvious, or even unassailably true? In developing answers to such questions, Thales began to make geometry *deductive.*

Pythagoras (530 B.C.)

Pythagoras and the school of thought that he founded, are of particular importance if one is to understand the development of science for the next 2000 years.

The Pythagoreans put forward the view that the aspects of the world that could be perceived with the senses, the world of direct experience, was imperfect (i.e. it was not as they personally thought it should be), and, unwilling to believe that God would create an imperfect universe, they turned, in the search for the "true" world, to an ideal world in their minds.

This spirit, of course, denies the validity of observations and experiments. It suggests that knowledge of the world should be arrived at through thought alone. Experimental science was deliberately rejected as being an untrustworthy way to study nature, for experiments dealt with real objects—things experienced through the senses. The study of pure geometry, however, involved *ideal* lines and curves. It was possible to obtain geometrical knowledge by pure thought, without the aid of observation or experiment. Accordingly, this branch of mathematics enjoyed a vigorous growth under the Pythagoreans.

The Athenians

We concentrate in the text material on the works of one of the Athenians, Aristotle. The contributions of other men who gathered at Athens during the period 500 to 300 B.C. are many. There were the philosophers, Socrates, Plato, Aristotle, and Epicurus;

the sculptor, Pheidias; the dramatists, Aeschylus, Sophocles, and Aristophanes; Herodus, the historian; scientists and mathematicians, such as Anaxagoras, Hippocrates, and Eudoxus. We shall look briefly at the ideas of two of the men who became known as the *Atomists.*

Leucippus and Democritus (440 B.C.)

The theory of the Atomists was based on a number of assumptions:

1. Matter is eternal, and no material thing can come from nothing, nor can anything material pass into nothing.
2. Material things consist of very minute, but not infinitely small, indivisible particles (the word "atom" meant "uncuttable" in Greek and, in discussing the ideas of the early Atomists, we could use the word "indivisibles" instead of the word "atoms").
3. All atoms are of the same kind, that is, of the same substance, but differ in size, shape, and position.
4. The atoms exist in an otherwise empty space (a void), which separates them, and because of this space they are capable of movement.
5. The atoms are in ceaseless motion although the nature and cause of the atomic motions are not clear. In the course of their motions atoms come together and form combinations which are the material substances we know. When the atoms forming these combinations separate, the substances break up. Thus, the combinations and separations of atoms give rise to the changes which take place in the world. The combinations and separations take place in accord with natural laws which are not known, but do not require the action of gods or demons or other supernatural powers.

Many of these proposals are similar or identical to those made by scientists centuries later, but the Greeks had no definite observed facts, nor the ability to test by experiment the consequences of these ideas. The Greek doctrine was founded on and incorporated into their general philosophical scheme of the world. It was thus very easy for it to be upset and replaced when a different philosophical scheme was proposed. Aristotle, arguing that no vacuum or void could exist, and that therefore the idea of atoms with their inherent motion must be rejected, did propose a different scheme and the ideas of these early Atomists almost completely died out.

The Alexandrian Period

Although Alexander the Great had little interest in science, he had a great respect for Greek culture and he spent huge sums of money in establishing at Alexandria in Egypt, a vast library which became a gathering place for the best scholars during the period 300 to 100 B.C. This period produced three astronomers and three mathematicians of the highest calibre. One of the mathematicians, Archimedes, has perhaps never had an equal.

Euclid (300 B.C.)

His first book, *The Elements*, synthesized knowledge of geometry and formalized the deductive system which we now call *Euclidean Geometry.*

Aristarchus (280 B.C.)

This astronomer outlined a geometrical analysis which yielded information about the relative sizes and distances of the sun and the moon. He showed that the sun was much more distant than the moon, and predicted that the sun's diameter would measure about twenty times that of the moon. (In fact, it is about four hundred times greater.) He also estimated the moon's diameter as being about half that of the earth. (The correct figure is about one-quarter.) His errors were due to inaccuracies of measurement, not errors in mathematics.

Eratosthenes (250 B.C.)

Eratosthenes determined a value for the diameter of the earth. His result was very nearly correct. He also suggested a calendar which became the basis of the Julian calendar which was used until the Gregorian calendar was introduced in 1582.

Archimedes (200 B.C.)

Archimedes was possibly the greatest mathematician of all time and he contributed to the fields of physics and engineering as well. In geometry, his work dealt mainly with areas and volumes of figures enclosed by curved lines and surfaces, such as, ellipses, parabolas, cones, and spheres. His best-known contribution to physics was his principle of flotation. In engineering, he put forth a theory of levers, pulleys, and screws.

Claudius Ptolemy (150 A.D.)

Using astronomical data compiled by Hipparchus, Ptolemy proposed a theory to account for the motions of stars and planets which has had the longest period of acceptance of any theory ever presented. More will be said of his system in Unit 2.

Diophantus (approximately 290 A.D.)

Diophantus developed a system of mathematics, begun centuries earlier by the Babylonians, into what we call algebra.

Chapter 2 *From the Complex to the Simple*

The Mathematics of Simple Motion

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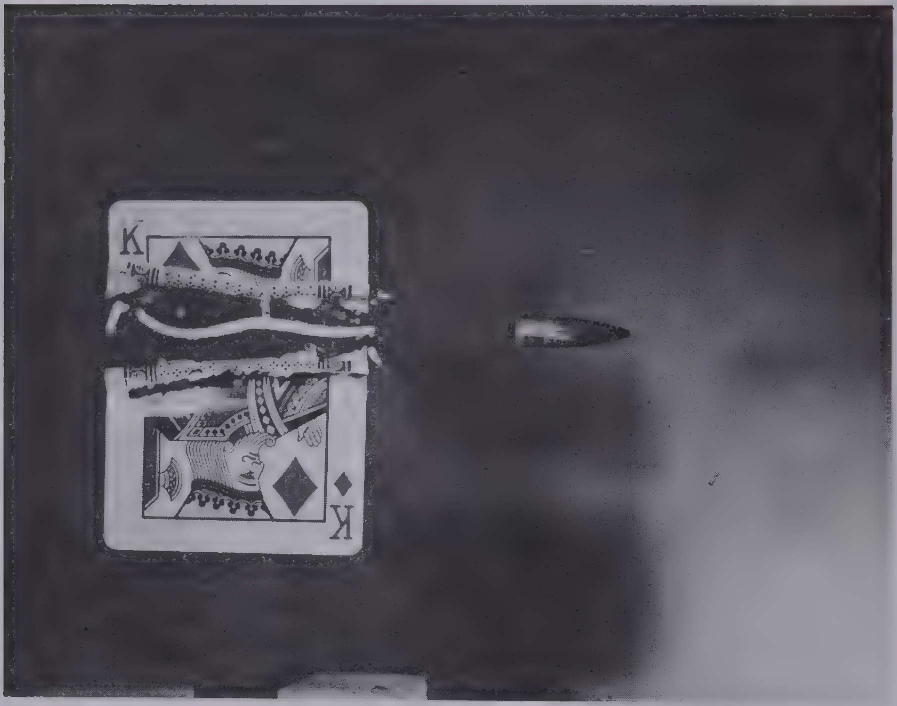


From the Complex to the Simple
The Mathematics of Simple Motion

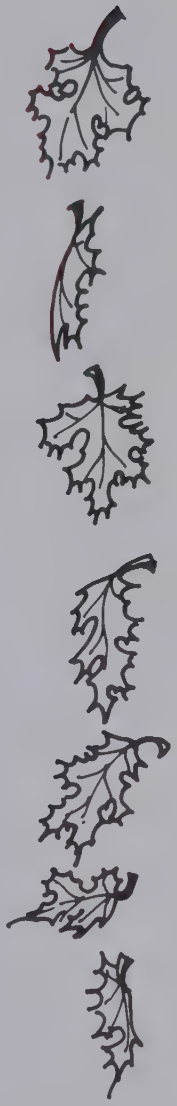
Chapter Two

2.1 The Study of Simple Uniform Motion

Man crawls, walks, runs, jumps, dances. To move himself faster, farther, higher, deeper, he invents things such as sleds, bicycles, submarines, and rocket ships. As human beings, we are caught up in motion and fascinated by it. Perhaps this is why so many artists try to portray movement. It is one reason why scientists investigate motion. The world is filled with things in motion: things as small as dust, as large as stars, and as common as ourselves; motion fast and slow, motion smooth, rhythmic, and erratic. We cannot investigate all of these at once. So from this swirling, whirling world of ours let us choose just one moving object for attention, something interesting and typical, and, above all, something manageable.

But where shall we start? We might begin our investigation by looking at a modern machine—the Saturn rocket, say, or a super-charged dragster, or an automatic washing machine. But as you know, things such as these, though made and controlled by man, move in very complicated ways. We really ought to start with something easier. Perhaps a bird in flight? Or a leaf falling from a tree?

Surely in all of nature there is no motion more ordinary than that of a leaf fluttering down from a branch. Can we describe how it falls or explain why it falls? As we think about it we quickly realize that, while the motion may be natural, it is very complicated: the leaf twists; turns; sails to the right and left, back and forth, as it floats down. Even a motion as ordinary as this may turn out, on closer examination, to be more complicated than that of machines. Although we might describe it in detail, what would we gain? No two leaves fall in quite the same way; therefore, each leaf would require its own detailed description. Indeed, this individuality is typical of many naturally occurring events on earth!



There is a very old maxim: "To be ignorant of motion is to be ignorant of Nature"

And so we are faced with a real dilemma. We want to describe motion, but the motions that excite and interest us appear to be hopelessly complex. What shall we do? We shall consider a very simple motion and attempt to describe it. Those of us who have learned to play a musical instrument will appreciate the wisdom of starting with simple tasks. If our music teacher had confronted us in our first lesson with a Beethoven piano sonata, we would in all probability have quickly foregone music in favour of a less taxing activity.

Before proceeding far into his study of motion Isaac Newton developed the branch of mathematics called calculus.

A modern scientist often uses the language of mathematics to describe natural phenomena. Mathematics provides concise and accurate methods of describing simple motion by graphs and algebraic equations. The use of mathematics to describe motion was introduced by scientists in the sixteenth century and since that time mathematics has gradually become an important tool of the scientist. In some cases developments in physics were delayed because the mathematics of the period was not adequate to analyze a particular phenomena of interest to the physicist. However in many other cases, mathematicians had developed the techniques and theories for use in mathematics many years before the scientists found that they could use them in their work. It is necessary in this chapter to learn some mathematical techniques and equations before we proceed further in our study of motion.

The mathematics that we shall develop describes some simple forms of motion. Science is often the study of an ideal situation that is seldom, if ever, achieved in the complex "real" world. The motion of an object travelling at constant speed in a straight line, although impossible to achieve in reality, is simpler to describe than that of a car travelling along a road. However, a study of the simpler situation often leads to a better understanding of the more complex one.

The format of this chapter is different from that of most of the other chapters in this book. You will find that as you read through the text you will be asked many questions. The answers to these questions appear in the margin opposite the question. Keep the margin covered as you work through these problems and use the answers as a check on your solution.

2.2 Uniform Motion: The Calculation of Distance Travelled

See Experiment 2.1, pg. 105.

We shall start by examining the simplest form of motion, *uniform motion*, which is defined as the *motion of an object travelling at constant speed in a straight line*. Consider a car travelling across the page to the right at a constant speed of 10 metres per second from the point x (as shown in Fig. 2.1).

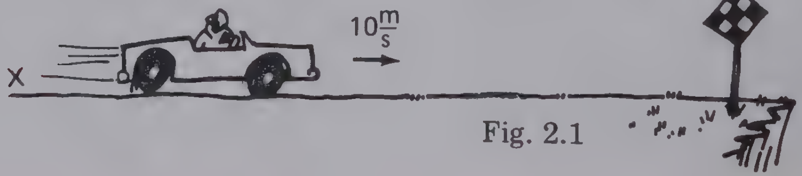


Fig. 2.1

1. How far would this car travel in 5 seconds?

A1 50 metres

2. What equation involving speed and time of travel did you use to calculate the distance travelled?

A2 distance travelled = speed x time-interval

3. Use this equation for describing uniform motion to calculate the distance travelled by the car after 7 seconds and after 50 seconds.

A3 70 m and 500 m

4. How long would it take this car to travel 100 m?

A4 10 s

An equation is often the most practical method of describing simple motion. We can use the equation of question two as our definition for speed.

$$\text{speed} = \frac{\text{distance travelled}}{\text{time interval}}$$

We have already used variations of this defining equation to solve some problems describing uniform motion. Another useful method to describe motion is to draw a graph.

time (t) seconds	distance (x) metres
0	0
5	50
10	100
15	150
20	200
25	250
30	300
35	350
40	400
45	450

Distance – Time Graph

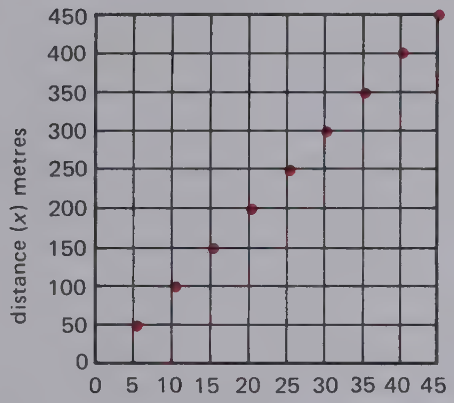


Fig. 2.2

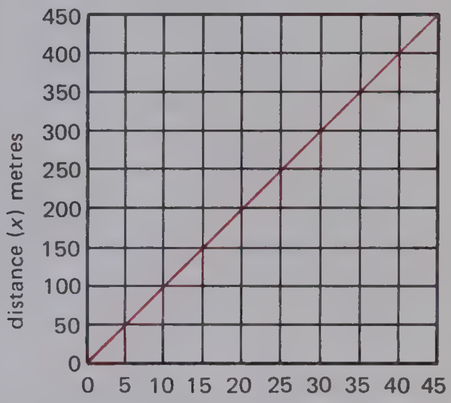


Fig. 2.2a)

The chart above shows the distance travelled by the car in Fig. 2.1 at 5 second intervals. The distance is measured from the starting point. These values are plotted on the distance-time graph Fig. 2.2. It is customary in drawing motion graphs to plot time on the horizontal axis. If we plot the distance-time graph for much smaller time-intervals than 5 seconds, the plotted points would form a straight line as shown in Fig. 2.2a. Use the graph in Fig. 2.2a to answer the following questions.

5. How far did the car travel from the starting point in 25 seconds?

A5 250 m

6. How far did it travel between the times of 20 and 40 seconds?

A6 200 m

7. How long did it take to travel a distance of 170 metres?

A7 17 s

Both equations and graphs are useful methods of describing motion. Some students prefer graphs because graphs let them visualize what is happening. Draw in your notebook the distance-time (x-t) graphs representing objects travelling in uniform motion at 5 metres per second, 20 metres per second, and 30 metres per second. Draw the three lines on the same set of axes

Now look at your graph and answer the following questions.

8. What features do the three lines on your graph have in common?
9. What are their differences?

A8 They are all straight lines and they pass through the origin.

A9 They each have different slopes.

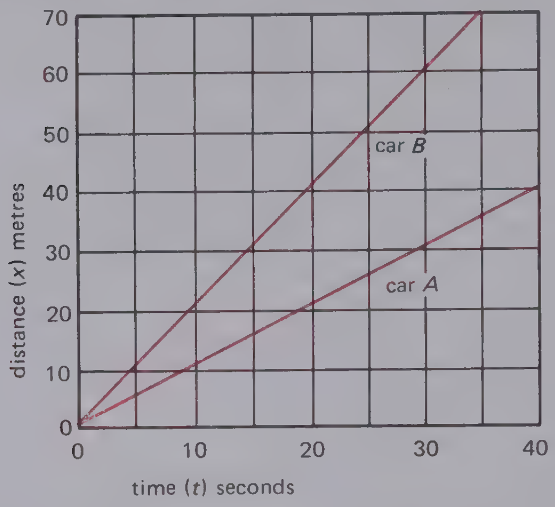


Fig. 2.3

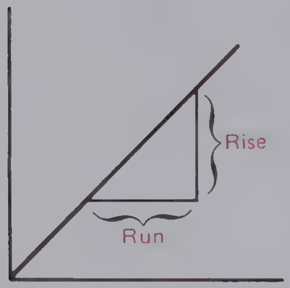
In Fig. 2.3 the uniform motion of two cars, *A* and *B*, is described by means of a distance-time graph. Examine the graph and answer the following questions. You might find it useful to refer to the graph you have drawn in your notebook.

10. Which car is travelling faster, *A* or *B*?
11. What are the speeds of car *A* and car *B*?

A10 *B* is travelling faster than *A*

A11 V_a = 1 metre per second.
V_b = 2 metres per second.

If you had difficulty determining the speed of the objects from the distance-time graph consider the following method. The useful feature in determining speed from the distance-time graph for an object in uniform motion is the *slope of the line.* You have seen, from the graph drawn in your notebook to answer questions 8 and 9, that the slope of the line representing a speed of 30 metres per second is steeper than the slopes of the lines representing 20 and 5 metres per second. The distance-time graph in Fig. 2.4 shows the uniform motion of the car travelling at a speed of 10 metres per second. Let us consider the distance travelled by the object after 10 seconds and after 25 seconds. During this 15 second time-interval (Δt), the object changes its distance from the starting point from 100 metres to 250 metres. The distance travelled (Δx) during this time-interval is 150 metres. The speed of the object is calculated by the formula,



$$\text{speed} = \frac{\text{distance travelled}}{\text{time interval}} = \frac{\Delta x}{\Delta t} = \frac{150\ \text{m}}{15\ \text{s}} = 10\frac{\text{m}}{\text{s}}.$$

On the graph, this formula represents the slope of the line. *Slope* = $\frac{\Delta x}{\Delta t}$ = *speed.* The units in which we calculate the slope are the units in which we measure speed. $\frac{\Delta x}{\Delta t} = \frac{\text{metre}}{\text{second}}$.

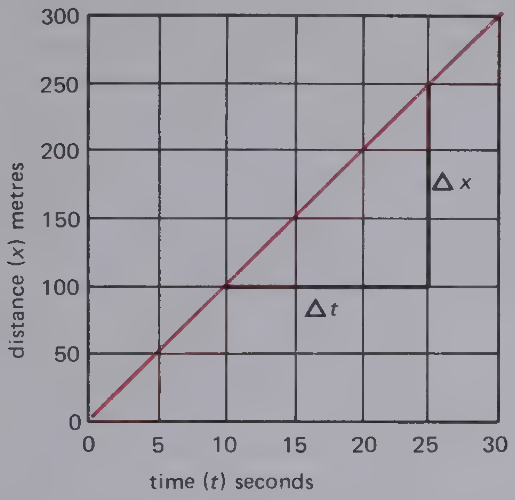
Δ(delta) is a mathematical symbol meaning "the change in". It is calculated by subtracting the initial value of a quantity from its final value.

$\Delta t = t_{final} - t_{initial}$
= time interval

$\Delta x = x_{final} - x_{initial}$
= distance travelled

Use this graphical method of determining speed to check your answers to questions 10 and 11.

Fig. 2.4



12. Does it matter which two points on the line you use to determine the slope? Try different pairs of points.

A12 You always get the same value for the slope regardless of the pair of points chosen.

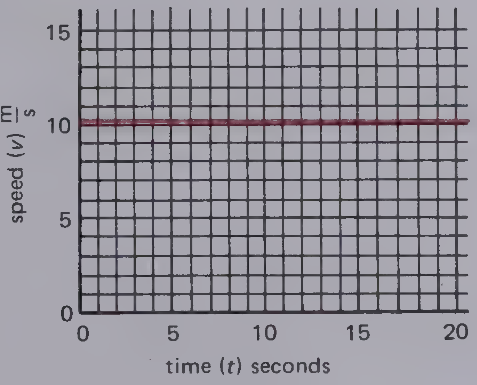
The slope of the line on a distance-time graph for an object moving with uniform motion equals the speed of the object.

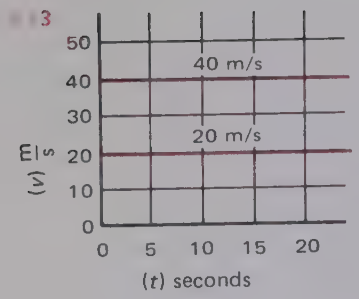
This fact illustrates an important feature of graphs. By considering other features of a graph in addition to the points plotted on it you can often determine additional information. Thus from a distance-time graph we can obtain information concerning the speed of the object.

2.3 Speed-Time Graph for Uniform Motion

A speed-time graph is another useful method of describing motion graphically. The speed-time graph (Fig. 2.5) represents the motion of the car travelling at a constant speed of 10 metres per second. The symbol v is commonly used to represent speed. The speed-time graph showing uniform motion is a straight horizontal line. The intercept of this line on the speed axis equals the speed.

Fig. 2.5





13. Describe the speed-time graphs representing uniform motions of 20 metres per second and 40 metres per second.

We have discovered that we are able to determine the speed of an object from its distance-time graph by considering the slope of the line. Can we calculate distance travelled from a speed-time graph? Consider Fig. 2.5. Can you see what feature of this graph would enable you to determine the distance travelled after 10 or 15 seconds? This is a difficult question. Remember, the application of graphical techniques to the study of motion was a brilliant leap of man's imagination.

Let us consider the distance travelled in 15 seconds by a car having a constant speed of 10 metres per second. The distance travelled may be calculated by the equation,

$$\Delta x = v \times \Delta t = 10 \text{ m/s} \times 15 \text{ s} = 150 \text{ metres.}$$

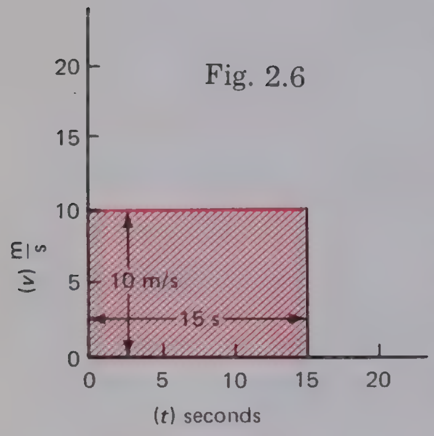


Fig. 2.6

In Fig. 2.6 we can see that the region of the graph under the line for the first 15 seconds of motion is rectangular in shape and has an area of 150 metres. Area under first 15 seconds of graph = length × width = 15 s × 10 m/s = 150 m. In a general case, the area under the line = $v \times \Delta t$ which numerically equals the distance travelled. Test this relation for some other time-intervals.

The area under the line on a speed-time graph for uniform motion equals the distance travelled (change in position) during the time-interval.

2.4 Summary for Uniform Motion

Definitions and Units

The defining equation for speed is

$$v = \frac{\Delta x}{\Delta t}$$

where Δx is the distance travelled and Δt is the time-interval. The units for measuring speed are always

$$\frac{\text{unit of distance,}}{\text{unit of time}} \quad \text{eg. metres/second, miles/hour.}$$

Uniform Motion is motion at a constant speed in a straight line.

Distance-Time Graph:

For uniform motion this graph is a straight line. The slope of the line equals the speed.

Speed-Time Graph:

For uniform motion this graph is a straight horizontal line. The intercept on the speed axis equals the speed. The area under the line for a certain time-interval equals the distance travelled during the time-interval.

The "Q" questions may be used to review some of the information discussed in the section 2.1 The answers are found on page 147.

Q1 State three different units that could be used to measure distance and speed. Use your imagination and make up some original ones.

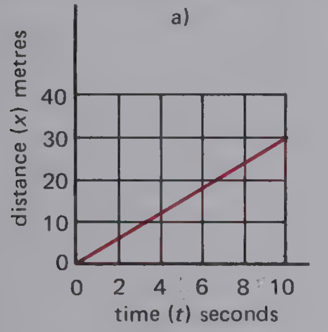
* **Q2** From among the examples below indicate those that illustrate uniform motion.

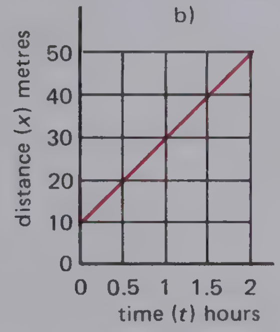
a) A billiard ball rolls on a flat table.
b) A ball is dropped out of a window.
c) An elevator travels upward in a very high building.
d) A football is thrown in the air and caught 30 metres away.
e) The sound of a shotgun blast travels across a field.
f) A sky diver falls freely for several hundred metres before pulling his ripcord.

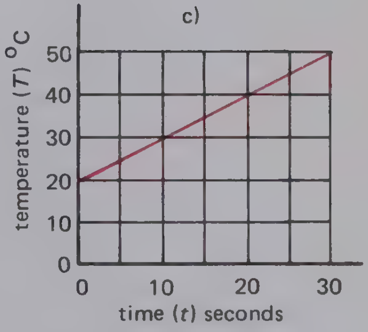
Q3 Use the problems in the chart below to review the equations.

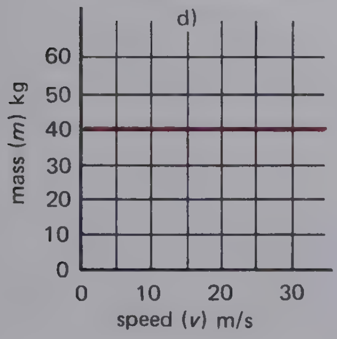
SITUATION		FIND
a	Speed uniform, distance travelled = 72 cm time elapsed = 12 s	Speed
b	Speed uniform at 45 kilometres per hour	Distance travelled in 20 minutes
c	Speed uniform at 36 m/min	Time to move 9.0 m
d	$x_1 = 0$ $t_1 = 0$ $x_2 = 15$ cm $t_2 = 5.0$ s $x_3 = 30$ cm $t_3 = 10$ s	Speed and distance travelled at 8.0 s

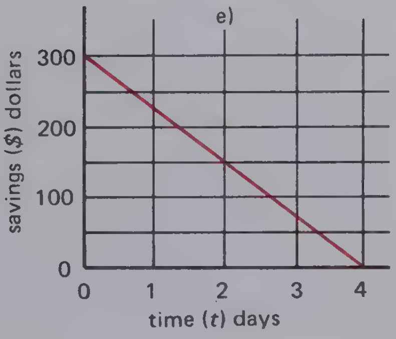
Q4 Determine the values for the slopes of the lines in each of the graphs below.

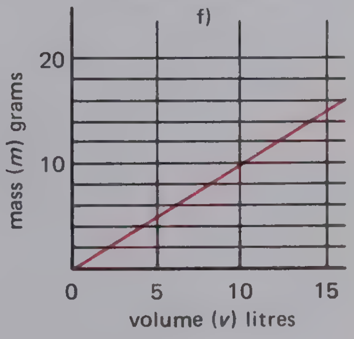




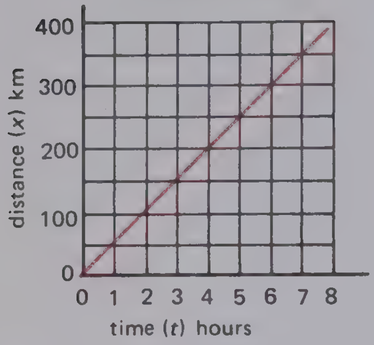


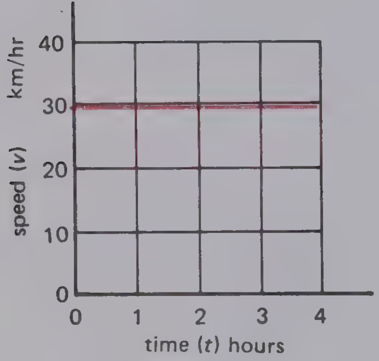






Graph for Q6





Q5

a) Sketch on the same axis of a distance-time graph lines showing the motions of objects travelling at constant speeds of 0.5 m/s, 2 m/s, and 5 m/s.

b) Check that the graphs are correctly drawn by comparing the slope of each line with the given speed.

c) Suggest an object that usually travels at each of the given speeds.

Q6 Examine the graph and answer the following questions:

a) How far did the object travel in 1, 2, and 5 hours?

b) What type of motion is described in the graph?

c) What was the speed of the object?

d) Describe the speed-time graph for this motion.

Q7

a) Describe the motion illustrated by the speed-time graph in the margin.

b) What graphical feature is considered when this graph is used to determine the distance travelled?

c) Use this feature to show the distance travelled in 3 hours.

See also problems 2.1 to 2.8 on page 28.

2.5 Uniform Acceleration—What is Acceleration?

The study of uniform motion is a useful method of introducing some graphical techniques that we shall use in our study of kinematics. However most objects do not travel at constant speed. They start, stop, speed up, or slow down. Or, in one word, they accelerate. We shall now extend our graphical methods by considering objects which are accelerating.

Acceleration is defined as *the rate of change of speed.* This definition may be written as the mathematical equation,

$$a = \frac{\Delta v}{\Delta t}.$$

If an object changes its speed from 10 metres per second to 30 metres per second in 5 seconds the rate of change of speed or acceleration is,

$$\frac{\Delta v}{\Delta t} = \frac{20\ \text{m/s}}{5\ \text{s}} = 4\ \frac{\text{m/s}}{\text{s}} \text{ or } 4\ \frac{\text{m}}{\text{s} \times \text{s}}.$$

See Experiment 2.2, pg. 108.

This is often written as $4\frac{\text{m}}{\text{s}^2}$ and it is read as 4 metres *per second per second.*

In reality, objects accelerate in a non-uniform manner. A dragster starting from rest experiences his highest acceleration at the beginning of the run. A rocket lifting off from a launching pad starts with a low acceleration which gradually increases as the fuel is burned up. In both these examples the speed of the object is increasing but the acceleration is also changing. In this section we shall limit our discussion to the study of uniformly accelerated motion. **Uniformly accelerated motion** *is a motion in which an object changes its speed at a constant rate.* If the object with an acceleration of 4 m/s^2 in our earlier example were accelerating uniformly, it would mean that its speed increased by 4 metres per second for each second that it was accelerating. Use the definition for acceleration to answer the following questions.

14. If an object starts from rest and accelerates uniformly at 8 metres per second per second, what would its speed be after 10 s, 12 s, and 15 s?

15. Determine the acceleration of an object which accelerates uniformly from a speed of 8 m/s to 64 m/s in 8 seconds.

A14 v = 80 m/s after 10 s
v = 96 m/s after 12 s
v = 120 m/s after 15 s

A15
$$a = \frac{\Delta v}{\Delta t} = \frac{(64-8)\ \text{m/s}}{8\ \text{s}} = 7\ \text{m/s}^2$$

2.6 Speed-Time Graph for Uniform Acceleration

If we want to describe the motion of an object which starts from rest and accelerates uniformly at a rate of 4 metres per second per second, we can use one of several methods.

(i) We may use a written description, such as the previous sentence.

(ii) We may also write the mathematical equation, $v = 4\,\Delta t$ metres per second. Where v represents the speed after a time interval Δt seconds.

(iii) We may use a chart like the one in the margin which shows the speed of the object at 5-second intervals of time.

t (s)	v (m/s)
0	0
5	20
10	40
15	60
20	80
25	100

A fourth possibility, and the one which we will now consider in some detail, is a graphical description. The speed-time graph in Fig. 2.7 describes the motion of the object starting from rest and accelerating at 4 metres per second per second. The speed-time graph of an object having uniform acceleration is a straight line. Let us analyze this graph using some of the techniques we acquired in our study of uniform motion.

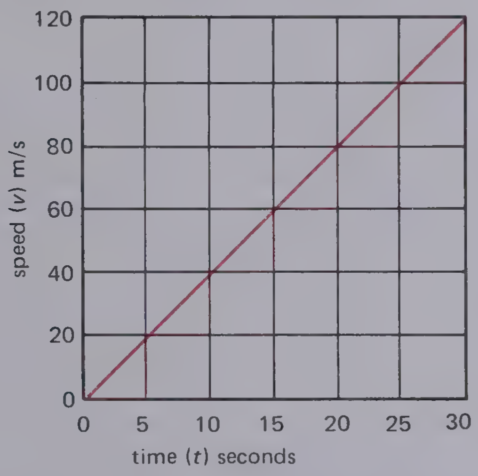


Fig. 2.7

16. What is the numerical value of the slope of this line?

17. What are the units of the slope?

18. What does the slope of this line represent?

19. Use this rule to determine the accelerations of the two objects whose motion is shown in the graph in Fig. 2.8.

A16 4

A17 $\frac{\text{m/s}}{\text{s}}$

A18 acceleration $\frac{\text{rise}}{\text{run}} = \frac{\Delta v}{\Delta t} = a$

A19 $a_y = 1.5\ \text{m/s}^2$
$a_x = 0.5\ \text{m/s}^2$

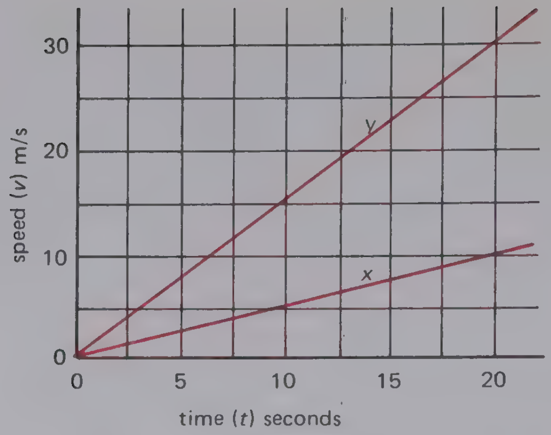


Fig. 2.8

The slope of the line on a speed-time graph of an object accelerating uniformly equals the acceleration.

The fact that the slope of the speed-time graph equals the acceleration can be recognized by considering the definition of acceleration. Acceleration is defined by the equation $a = \frac{\Delta v}{\Delta t}$. This is also the equation of the slope of the line on the speed-time graph. Slope $= \frac{\text{rise}}{\text{run}} = \frac{\Delta v}{\Delta t}$. Similarly we can show that the slope of the distance-time graph, which we used in section 2.3 to represent the speed, is recognized as another way of stating the definition of speed. Speed is defined by the equation $v = \frac{\Delta x}{\Delta t}$ and this equation equals the slope of the line on the distance-time graph.

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}\ .$$

In the case of uniform motion, the area under the line on the speed-time graph was shown to be numerically equal to the distance travelled. This is also true for an object travelling with uniform acceleration. We shall not prove this statement in this course but, because it provides a useful method of calculating the distance travelled, you are asked to accept it as true. It will be proved in a later course of physics or as part of a calculus course in mathematics.

The area under the line on a speed-time graph for a uniformly accelerating object equals the distance travelled during the time-interval.

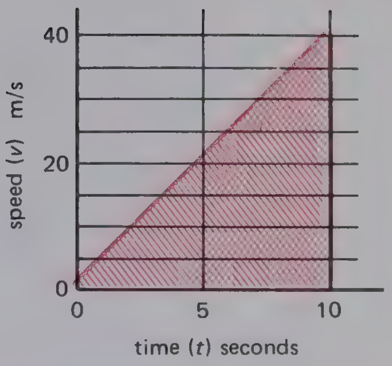
A20 $\Delta x = 200$ m

20. Use this fact and the graph in Fig. 2.7 to determine the distance travelled during the first 10 seconds of travel for the object accelerating uniformly at 4 metres per second per second.

Did you have difficulty with question 20? Consider this. The distance travelled during the first 10-second interval is the area under the graph for this time-interval. As shown in the marginal sketch the shape of this area is a triangle. Therefore the distance travelled,

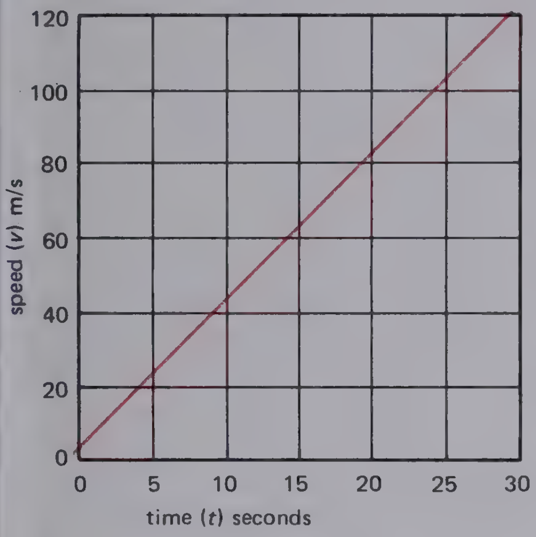
$$\begin{aligned} \Delta x &= \text{area of triangle} \\ &= \tfrac{1}{2}\ (\text{base} \times \text{height}) \\ &= \tfrac{1}{2}\ (10\ \text{s})\ (40\ \text{m/s}) \\ &= 200\ \text{metres.} \end{aligned}$$

21. Use this technique to determine the distance travelled after 5 seconds, 15 seconds, and 25 seconds of travel.



A21 Δx = 50 m, 450 m, and 1250 m after 5, 15, and 25 s respectively.

The chart and distance-time graph below (Fig. 2.9) were drawn by using information on the speed-time graph, to illustrate the uniformly accelerating motion which we have been discussing. We have placed the speed-time graph beside them for comparison.



time seconds	distance metres
0	0
5	50
10	200
15	450
20	800
25	1250

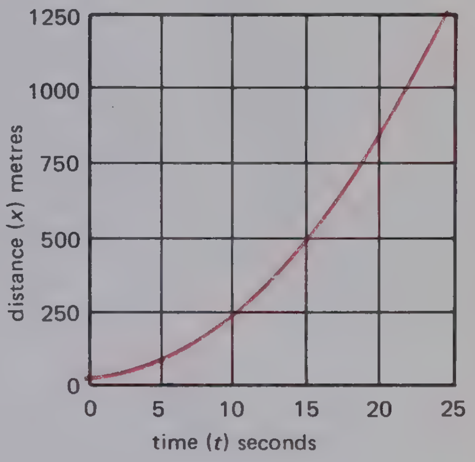


Fig. 2.9

Summary of Speed-Time Graph for Uniform Acceleration

> *The speed-time graph for uniformly accelerated motion is a straight line. The slope of this line equals the acceleration. The area under the line for a certain time-interval equals the distance travelled during that time-interval.*

* Q8 State five different sets of units that could be used to measure acceleration.

Q9 Determine the acceleration in each of the following situations. (Assume that the acceleration is uniform in each case.)

a) An object starting from rest rolls down a ramp and reaches a speed of 40 metres per second in 6 seconds.

b) A car travelling along a flat road maintains a speed of 60 kilometres per hour for 10 minutes.

c) A car travelling at 30 metres per second applies the brakes and stops in three seconds.

d) An object dropped from a window attains a speed of 38 metres per second after 4 seconds.

e) A bus travelling at 15 kilometres per hour accelerates to 30 kilometres per hour in 30 seconds.

Q10 A runner is able to accelerate at 3 metres per second per second from a standing start. What speed will he reach after 0.5 seconds?
If he continues at this acceleration how long would it take him to reach a speed of 30 metres per second? Does your answer seem realistic?

Q11 The speed-time graph in Fig. 2.11 shows the motion of two uniformly accelerating objects.

a) What is the acceleration of each of the objects?

b) How far does each object travel in 5 seconds?

Q12 The speed-time graph in Fig. 2.10 also describes the motion of a uniformly accelerating object.

a) How does the motion of this object differ from those described in Q11?

b) How far does it travel in 5 seconds?

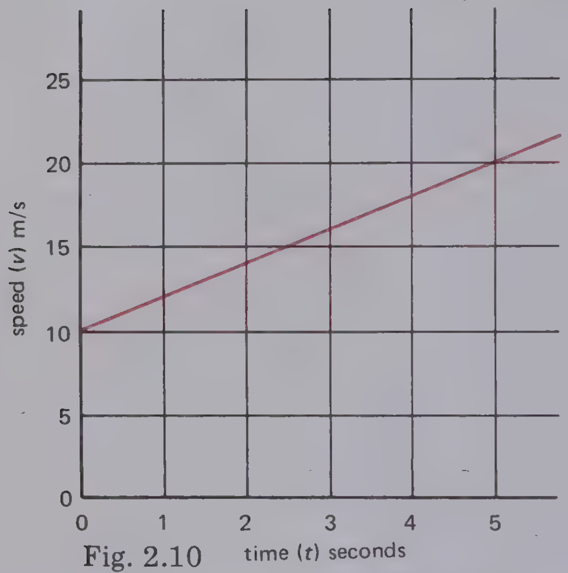


Fig. 2.10

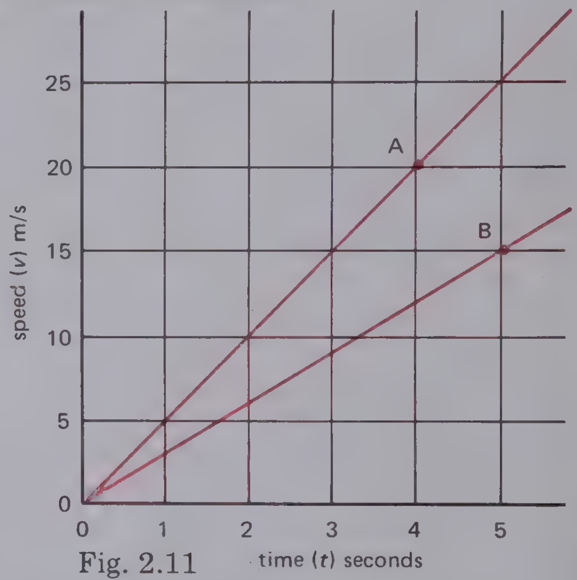


Fig. 2.11

See also questions 2.9 to 2.13 on page 28.

2.7 Instantaneous and Average Speed

In our discussion of uniform motion which considered objects travelling with constant speed, we saw that the speed was equal to the slope of the straight line on the distance-time graph. The analysis of a distance-time graph for uniformly accelerated motion is somewhat more complicated.

Examine the distance-time graph for uniformly accelerated motion in Fig. 2.12.

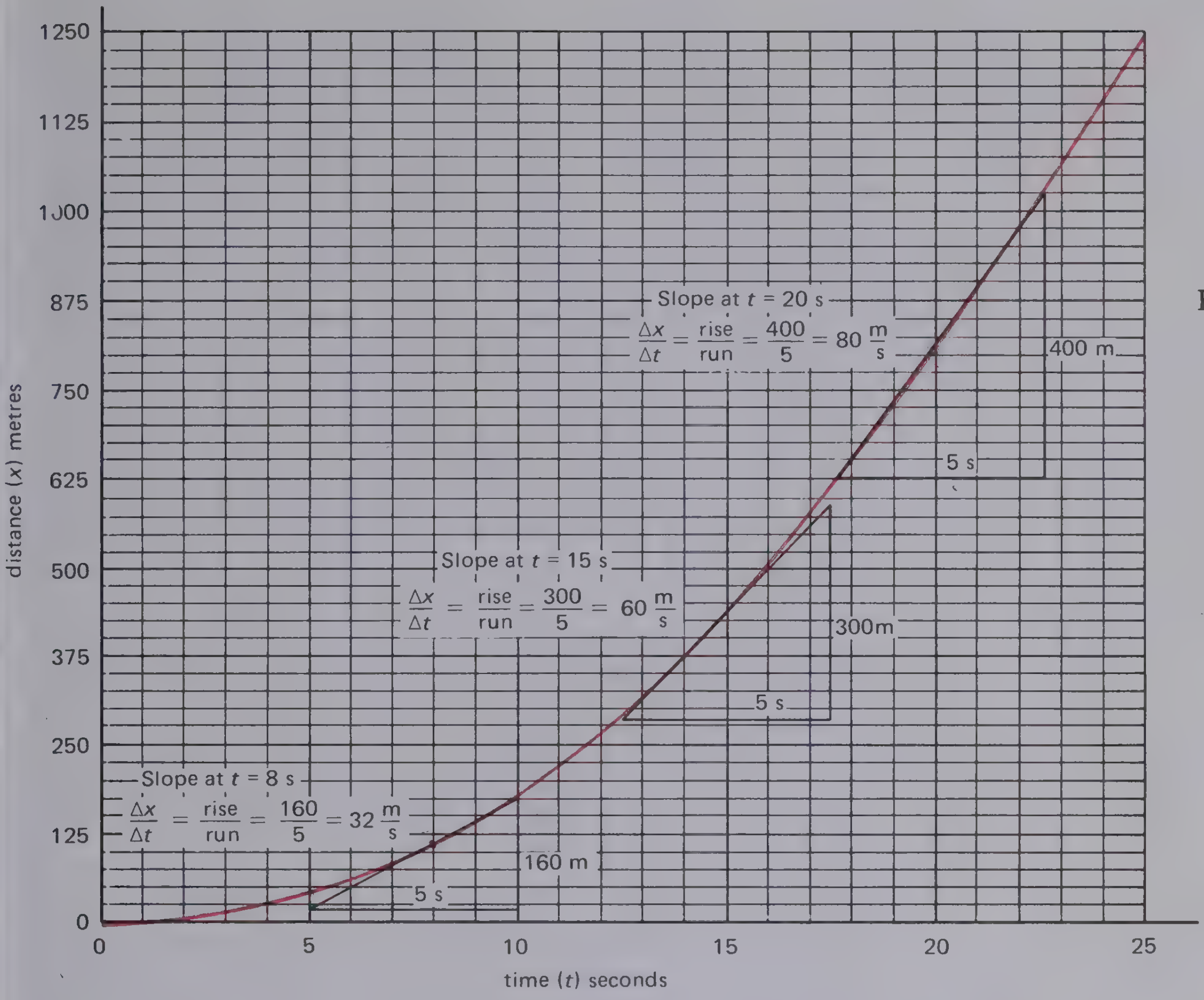


Fig. 2.12

22. What is the slope?
23. What is the speed?

A22 The slope is not constant. It increases as time increases.

A23 It is not constant. It is different at different times.

In order to answer questions 22 and 23 we must determine a method to calculate the slope of a curve at a particular point. Mathematicians have proved that the slope of a curve at any point is equal to the slope of the *tangent* drawn at that point. In Fig. 2.12 we have drawn three tangents to the curve on the distance-time graph at 8, 15, and 20 seconds. Upon comparison with the values of the speed in the speed-time chart (Fig. 2.9) for 8, 15, and 20 seconds we observe that they are equal.

The slope of the tangent at these points equals the instantaneous speed at a given time. Speed when given for a particular instant in time is called the **instantaneous speed**. In practice it is very difficult to measure instantaneous speed. The equation which defines speed is $v = \frac{\Delta x}{\Delta t}$. It indicates that we should measure the distance travelled during a time-interval Δt. In order to measure the speed at a particular instant in time, one tries to make the time Δt as small as possible. This is what we are attempting to do when we draw the tangent to the curve.

Tangent: a straight line touching a curve at only one point.

The shape of this distance-time curve is called a parabola.

A24 Speed is 20, 40, and 92 metres per second after 5, 10, and 23 seconds respectively.
Check: after 5 seconds $\Delta v = a\Delta t = 4$ metres per second^2 × 5 seconds = 20 metres per second.

24. Determine the instantaneous speed for the motion described in Fig. 2.12 at 5, 10, and 23 seconds by estimating the slope of the tangent to the curve at these points. Check your values by using the formula $\Delta v = a\Delta t$.

We can use the graph in Fig. 2.12 to describe another use of the word speed. The object, whose motion is described in the graph, travelled a total distance of 800 metres during the first 20 seconds of its motion. *Its average speed* for this time period = $\frac{\text{800 metres}}{\text{20 seconds}}$ = 40 m/s. This does not mean that the object was always travelling at a speed of 40 m/s. (In fact, we know the speed changed from 0 to 80 m/s during the 20 seconds.) However, it is interesting to note that if the object did travel at a constant speed of 40 m/s it would have travelled the 800 metres in the same period of time. **Average speed** *during a time-interval is defined as the total distance travelled divided by the time taken.* The equation $v_{av} = \frac{\Delta x}{\Delta t}$ measures the average speed during the time interval Δt. However, if we make Δt very small, we measure the average speed over a very small time-interval and, in the limit, at a particular instant in time.

25. What is the average speed for a car which travels 40 kilometres in one-half hour?

26. What is the average speed for the object whose motion is described in Fig. 2.12 for the first 25 seconds of its journey?

A25 $80 \frac{\text{kilometres}}{\text{hour}}$

A26 50 metres per second

Summary of Instantaneous and Average Speed

Instantaneous Speed: is the speed of an object at a particular instant in time. It is numerically equal to the slope of the tangent to the curve on a distance-time graph. It may be determined by using the equation $v = \frac{\Delta x}{\Delta t}$ *where* Δt *is made as small as possible.*

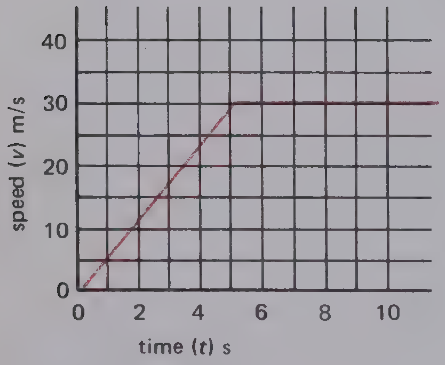
Average Speed During a Time-Interval: is the total distance travelled divided by time taken. It is defined by the equation

$$v_{av} = \frac{\Delta x}{\Delta t}$$

Q13 For Rally Drivers

A driver in a car rally travels from Toronto to Vancouver, a distance of 3200 kilometres (2000 miles), in 50 hours. Calculate his average speed. If he could maintain this average speed, how long would it take him to circumnavigate the earth at the equator? (Radius of the earth is 6400 kilometres.)

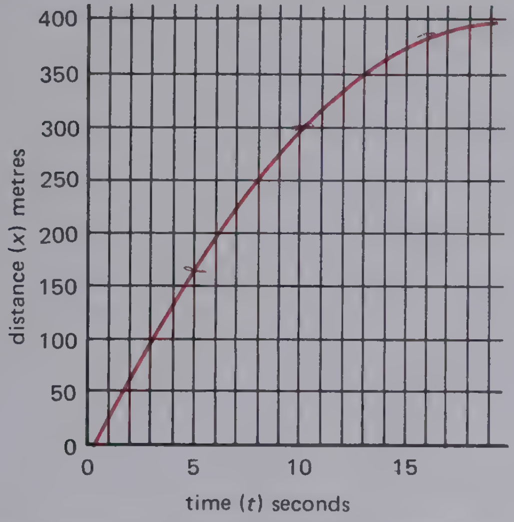
Q14 The speed-time graph in the margin describes the motion of a car which starts from rest. Use this graph to determine the distance travelled by the car in 10 seconds. Determine also the average speed for the first 10 seconds of this trip.



Q15 Examine the distance-time graph in Fig. 2.13.

a) Describe the illustrated motion in a few sentences.
b) Determine the average speed for the 20-second trip.
c) Estimate the instantaneous speed after 5 and 15 seconds.
d) Is this an example of uniform acceleration?

Fig. 2.13



See also questions 2.14 to 2.16 on page 29.

2.8 Derivation of Algebraic Equations for Uniform Acceleration

As we have seen, graphs are a very useful method of describing simple motion. They permit us to visualize the motion and, by means of analysis using slope and area, permit us to determine characteristics of the motion that are not readily visible at a first glance. We have also used some mathematical equations to describe simple motion. For uniform motion, we use the equation $v = \frac{\Delta x}{\Delta t}$ to determine the speed of an object. Acceleration was defined by the equation $a = \frac{\Delta v}{\Delta t}$. The use and study of equations is part of a branch of mathematics called *algebra.* Algebra has become a most useful tool of the scientist, who can often use equations to describe natural phenomena. In the final analysis, the language of nature is mathematics. In order to describe uniform acceleration more completely we are going to develop some additional equations. These equations will be derived using graphical techniques.

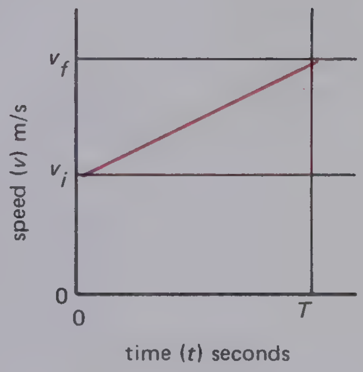
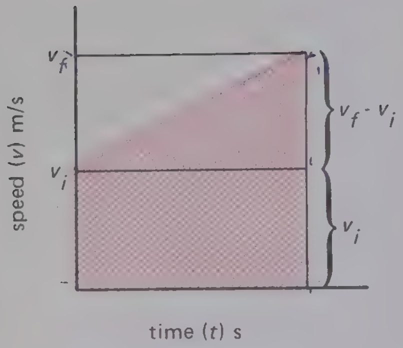


Fig. 2.14

Consider the motion described by the speed-time graph in Fig. 2.14. This graph shows the motion of an object which starts with an initial speed v_i and then accelerates uniformly to a final speed v_f. The time-interval for this change is T ($\Delta t = T$). From our study of motion graphs we know that the area under the line for uniform acceleration represents the distance travelled during the time-interval. We will use the symbol d to represent the distance travelled during the time-interval, ($d = \Delta x$).

Distance travelled = d = area under the line for the time-interval T. This area is indicated in the graph in the margin. It



may be conveniently divided into two parts, a rectangle and a triangle.

Therefore,
$$\begin{aligned} d &= \text{area of rectangle} + \text{area of triangle} \\ &= v_iT + \tfrac{1}{2}\,(v_f - v_i)\,T \\ &= \tfrac{1}{2}\,v_iT + \tfrac{1}{2}\,v_fT \\ &= \frac{(v_i + v_f)}{2}\,T \end{aligned}$$

Equation # 3
$$d = \left(\frac{v_i + v_f}{2}\right)T.$$

It is useful to examine the terms that make up this equation. In our discussion of average speed we used the equation, distance travelled = average speed × time-interval. The equation, $d = \left(\frac{v_i + v_f}{2}\right) T$ represents the same thing. The average speed for an object experiencing uniform acceleration is the average of the initial and final speeds. $v_{av} = \frac{v_i + v_f}{2}$ for uniform acceleration.

If the object is initially at rest ($v_i = 0$) the equation simplifies to

$$d = \left(\frac{v_f}{2}\right)T$$

It is useful to remember that the average speed for uniformly accelerated objects is simply the average of the initial and final speeds.

Example Problem

Consider the following problem. A car travelling at 20 metres per second accelerates uniformly to 45 metres per second in 20 seconds. Determine the distance travelled during this change in speed. The given information might be summarized as follows:

$$v_i = 20\ \text{m/s},\ v_f = 45\ \text{m/s, and}\ T = 20\ \text{s}$$
$$\begin{aligned} d &= \frac{v_i + v_f}{2}\,T \\ &= \frac{20 + 45}{2}\,\frac{\text{m}}{\text{s}} \times 20\ \text{s} \\ &= 32.5\ \frac{\text{m}}{\text{s}} \times 20\ \text{s} \\ &= 650\ \text{m}. \end{aligned}$$

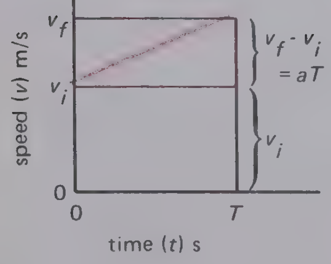
The distance travelled is 650 metres.

27. While passing another car, a driver accelerates uniformly from 18 m/s to 30 m/s in 12 seconds. Determine the passing distance for this car.

A27 288 metres

Another useful equation may also be derived by considering again the motion described in Fig. 2.14. This graph has been drawn again in Fig. 2.15 to emphasize the features shown in this derivation. The distance travelled during the time-interval is equal to the area under the line, which as before, may be considered as a rectangle plus a triangle. The height of the triangle = $v_f - v_i$. But $v_f - v_i = \Delta v = aT$. Therefore the area of the triangle = $\frac{1}{2}$ (base × height) = $\frac{1}{2}$ $(T \times aT) = \frac{1}{2}\ aT^2$. This can be used to give us an alternative equation to calculate the distance travelled.

Fig. 2.15



Distance travelled,

$$d = \text{area of rectangle} + \text{triangle}.$$

$$= v_iT + \tfrac{1}{2}aT^2$$

Equation #4 $\quad d = v_iT + \frac{1}{2}aT^2.$

If the object starts from rest, $v_i = 0$, Equation #4 simplifies to $d = \frac{1}{2}aT^2$.

This equation consists of the two terms v_iT and $\frac{1}{2}aT^2$. The term v_iT equals the distance the object travels if it does not accelerate. The term $\frac{1}{2}aT^2$ equals the additional distance it travels due to the acceleration. The use of this equation may be illustrated by the following.

Example Problem

Determine the acceleration of a car which has an initial speed of 10 m/s and travels a distance of 30 m in 2 seconds. (It is apparent that the car must be accelerating because if it had continued at its initial speed of 10 m/s it would have taken 3 seconds to travel 30 m.)

The given information may be summarized as follows:

$v_i = 10$ m/s, $d = 30$ m. $T = 2$s, and $a = ?$

$$d = v_iT + \tfrac{1}{2}aT^2$$

$$30\text{m} = (10\text{ m/s} \times 2\text{s}) + \tfrac{1}{2}a\,(2\text{s})^2$$

$$30\text{m} = 20\text{m} + 2a\text{ s}^2$$

$$10\text{m} = 2a\text{ s}^2$$

$$a = \frac{10}{2}\frac{\text{m}}{\text{s}^2} = 5\frac{\text{m}}{\text{s}^2}$$

The acceleration of the car is 5 m/s^2.

Examine below the two equations that we have derived to calculate the distance travelled during uniform acceleration.

Equation #3, $\quad d = \left(\frac{v_i + v_f}{2}\right)T$

Equation #4, $\quad d = v_iT + \frac{1}{2}aT^2$

Your choice of a particular equation to solve a problem will depend upon the information given in the problem. These two equations or combinations of them may be used to solve a wide variety of problems involving motion with uniform acceleration. There are problems at the end of this section to give you some experience in using these equations.

2.9 Summary of Equations Used in Studying Simple Motion

Symbols used in equations:

$d = \Delta x =$ distance travelled

$T = \Delta t =$ time interval

$v_i =$ initial speed

$v_f =$ final speed

$a =$ constant acceleration

Uniform Motion:

$$1.\ v = \frac{\Delta x}{\Delta t} = \frac{d}{T}$$

Motion with uniform acceleration:

$$2.\ a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{T}$$

$$3.\ d = \left(\frac{v_i + v_f}{2}\right)T$$

$$4.\ d = v_iT + \frac{1}{2}aT^2$$

Now let's stop for a moment to assess what we have done. We have seen that distance-time graphs and speed-time graphs are useful methods to describe motion. The graphical techniques we have acquired permit us to obtain much information from these graphs. We have also used motion graphs to develop some algebraic equations to describe simple motion. To some of you the development of the equations to describe motion with constant acceleration may have appeared difficult and confusing. The reason they are included in the text is to show that these useful equations can be derived mathematically. These equations should be considered as tools to be used. Although many of you might have enjoyed the excursion into algebra, the method of their derivation is not of primary importance. What is important is that you use these equations to simplify your work in describing and solving problems of simple motion. The more you use these equations the better you will understand them.

Q16. The problems in the chart below will give you an opportunity to use algebraic equations to describe simple motion.

SITUATION	FIND
a) Object starts from rest and accelerates uniformly at 4 m/s^2 for 10 s.	Speed after 10 s.
b) Initial speed is 8 m/s and acceleration of 5 m/s^2 takes place for 3 s.	Distance travelled during 3 s. Speed after 3 seconds
c) Initially at rest an object accelerates uniformly to a speed of 50 m/s in 30 seconds.	Acceleration Distance travelled
d) Object travelling at 20 m/s accelerates to 40 m/s over a distance of 100 metres.	Time-interval of acceleration
e) Object accelerates uniformly from rest travelling a distance of 50 m in 4 seconds.	Acceleration Final speed after 4 s

See also problems 2.17 to 2.21 on page 29.

.10 Summary for Uniformly Accelerated Motion

Definitions and Units

Acceleration is the rate of change of speed.
The defining equation for acceleration is

$$a = \frac{\Delta v}{\Delta t} \quad \text{Equation \#2}$$

The Units for measuring acceleration are always

$$\frac{\text{Unit of speed}}{\text{Unit of time}} \quad \text{eg: } \frac{\text{m/s}}{\text{s}}, \frac{\text{m}}{\text{s}^2}, \frac{\text{mph}}{\text{s}}, \frac{\text{cm/hr}}{\text{s}}.$$

Uniform Acceleration occurs when an object changes its speed at a constant rate.

Speed-Time Graph:
For uniform acceleration this graph is a straight line.
The slope of the line equals the acceleration.
The area under the line equals the distance travelled (change in position).

Position-Time Graph:
For uniform acceleration this graph is a smooth curve (a parabola).
The slope of the tangent at any point on this curve equals the nstantaneous speed at that time.

Algebraic equations for calculation of distance travelled.

$$d = \left(\frac{v_i + v_f}{2}\right)T \qquad \text{Equation \#3}$$

$$d = v_iT + \tfrac{1}{2}\,aT^2 \qquad \text{Equation \#4}$$

Average speed is defined as the total distance travelled divided by the time interval $v_{av} = \frac{\Delta x}{\Delta t}$. In the case of uniform acceleration,

$$\text{average speed} = \frac{v_i + v_f}{2}$$

Problems Chapter 2

Section A

2.1 At a cruising speed of 225 km/hr, with a slight wind, a light aircraft consumes 45 litres (10 gallons) of fuel per hour. What range does the plane have if its fuel tank contains 225 litres of gasoline?

2.2 Estimate your speed as you walk from home to school. How long does the walk take? Use the relationship between speed, distance, and time to estimate the distance between your house and school? Can you check this distance by another method? Which is the more reliable?

2.3 The record for the highest puck speed is held by Bobby Hull whose left-hand slap shot has been measured at 52.5 m/s (118 miles/hour). If the reaction time of a goaltender is 0.25 seconds, from what least distance can Hull shoot without giving the goaltender enough time to react?

2.4 A tsunami, incorrectly called a tidal wave, was caused by an earthquake off the coast of Alaska in 1946. This tsunami consisted of several sea waves which travelled at an average speed of 800 km/hr (500 miles/hour). The first of the waves reached Hawaii 4 hours and 34 minutes after the earthquake occurred. From this data, calculate how far the origin of the tsunami was from Hawaii.

2.5 A bus driver's log for a 450-kilometre trip is shown in the chart.

	Distance from	
Place	Start (km)	Time
Atown	0	12:15 pm
Bville	80	1:15 pm
Cberg	100	1:45 pm
Dplace	160	2:30 pm
Etown	250	3:30 pm
Fville	320	4:15 pm

a) What was the average speed of the bus from Atown to Fville?
b) If the bus had continued throughout the entire trip with its average speed from Atown to Bville, at what time would it have arrived at Fville?
c) What was the highest speed reached by the bus during the trip?
d) Plot a distance-time graph for this journey and use it to determine the average speed from Cberg to Dplace and from Dplace to Etown.

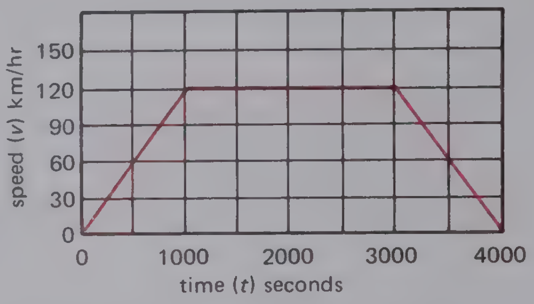
2.6 The electron beam in a typical TV set sweeps out a complete picture in 1/30 seconds and each picture is composed of 525 lines. If the width of the screen is 50 cm (20 inches), what is the speed of that beam over the surface of the screen?

2.7 Car *A* and car *B* are travelling along the same highway in the same direction. Car *B* is located 30 km in front of car *A*. Car *A* is travelling at 80 km/hr and car *B* at 50 km/hr. Sketch a distance-time graph to show when car *A* will overtake car *B*.

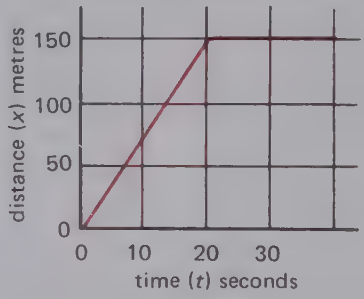
2.8 Two planes leave the same airport at 3:00 pm to fly to the same distant city.
One has a speed of 480 km/hr and the other has a speed of 800 km/hr. How long will it take the faster plane to get 480 km ahead of the slower one?

2.9 The graph below describes the motion of a train on a journey from Xville to Ytown. From the graph determine the following:

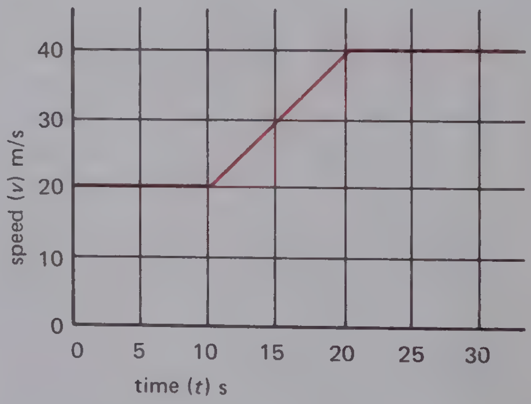
a) the highest speed reached by the train.
b) the acceleration at the beginning of the trip.
c) the distance from Xville to Ytown.
d) the average speed of the train for this journey.



2.10 Draw the speed-time graph for the motion described by the distance-time graph below.



2.11 Draw the distance-time graph for the motion described by the speed-time graph following.



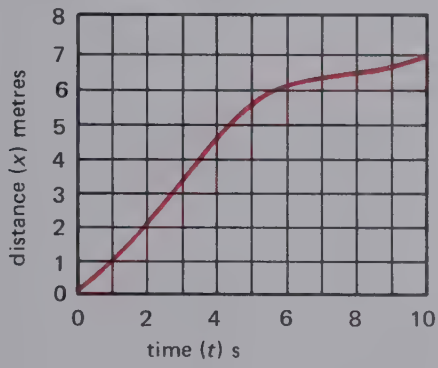
.12 In drag racing a dragster races over a 440-yard istance from a standing start. The record for lowest me for this run with a piston engine is 6.3 seconds inal speed 228.42 miles per hour). his record is held by Sarge Arciero of Broomal, ennsylvania, driving a 392 cubic inch Chrysler AAF April 1971. /hat was the average acceleration of the run? (Determine answer in mph/s.)

.13 Sketch graphs for the following situations.
) Distance-time graph for an object travelling at a onstant speed of 30 metres per second.
) Speed-time graph for an object starting at 15 netres per second and accelerating at 5 metres per econd per second.
) Speed-time graph for an object starting at 18 netres per second and slowing down at a rate of 2 netres per second per second.
) Distance-time graph for the object described in).

.14 If you travelled one kilometre at a speed of 1000 m per hour and another kilometre at a speed of km per hour, your average speed would not be $\frac{000 \text{ km/hr} + 1 \text{ km/hr}}{2}$ or 500.5 km/hr. What would e your average speed? (Hint: What is the total disance and total time?)

.15 What is your average speed in each of these ases?
) You run 100 metres at a speed of 5.0 metres er second and then you walk 100 metres at a speed f 1.0 metres per second.
) You run for 100 seconds at a speed at 5.0 metres per second and then you walk for 100 seconds at speed of 1.0 metres per second.

.16 Careful analysis of a stroboscopic photograph of moving object yielded information which was plotted on the graph below. By placing your ruler tanent to the curve at appropriate points, estimate the ollowing:
) At what moment or in which interval was the peed greatest? What was the value of the speed at hat time?
) At what moment or in which interval was the peed least? What was it at that time?
) What was the speed at time t = 5.0 seconds?
) What was the speed at time t = 0.5 second?
) How far did the object move from time t = 7.0 econds to t = 9.5 seconds?



2.17 A toy car accelerates by means of a small rocket-type engine. If the acceleration during the burn is 5 metres per second per second and the burn lasts 3 seconds, determine the speed and the distance travelled at the end of the burn.

2.18 Show that the equation $d = v_iT + \frac{1}{2}aT^2$ is dimensionally correct, by showing that the units of the right-hand side are the same as the units on the left.

2.19 The data in the chart was taken from the acceleration graph for the Citroen SM on the page following. It was necessary to convert the units on the graph. Use this data to determine the average acceleration and distance travelled for the run from standing start to speeds of 10, 20, 30, and 40 metres/seconds. Use the appropriate algebraic equation and assume uniform acceleration to determine the distance.

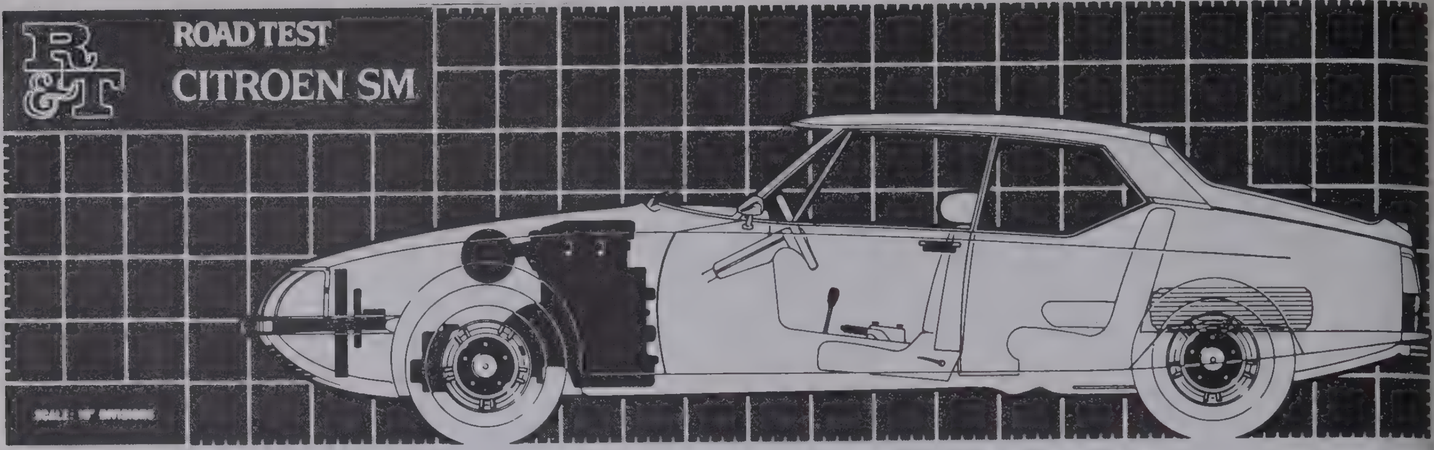
Speed Change (metres per second)	Time from Standing Start (seconds)
0 — 10	3.0
0 — 20	6.3
0 — 30	11.9
0 — 40	23.1

2.20 A car travelling at 40 m/s accelerates uniformly to 60 m/s. Determine the average speed if the time of acceleration was 5 seconds. What would the average speed be if the time-interval of the acceleration was 10 seconds?

2.21 Use the given data and the appropriate algebraic equations to answer the questions in the chart below. (Assume uniform acceleration in each case).

	Given Data	Find
a	v_i = 8 m/s	a = ?
	v_f = 20 m/s	d = ?
	T = 3 s	
b	v_i = 20 m/s	d = ?
	T = 10 s	v_f = ?
	a = 4 m/s^2	
c	d = 1000 m	a = ?
	v_i = 30 m/s	v_f = ?
	T = 20 s	
d	d = 45 m	T = ?
	v_i = 0	a = ?
	v_f = 30 m/s	
e	a = 10 m/s^2	T = ?
	v_i = 30 m/s	d = ?
	v_f = 100 m/s	

ROAD TEST
CITROEN SM



ENGINE

Type	dohc 90° V-6
Bore x stroke, mm	87.0 x 75.0
Equivalent in	3.42 x 2.95
Displacement, cc/cu in	2670/163
Compression ratio	9.0:1
Bhp @ rpm	180 @ 6250
Equivalent mph	142
Torque @ rpm, lb-ft	172 @ 4000
Equivalent mph	90
Carburetion	three Weber 42DCNF (2V)
Type fuel required	premium, 98-oct
Emission control	none on test car

GENERAL

Curb weight, lb	3270
Test weight	3640
Weight distribution (with driver), front/rear, %	61/39
Wheelbase, in	116.1
Track, front/rear	60.1/52.2
Overall length	192.6
Width	72.3
Height	52.1
Ground clearance	(normal) 6.1
Overhang, front/rear	39.5/37.0
Usable trunk space, cu ft	9.4
Fuel tank capacity, U.S. gal	19.7

DRIVE TRAIN

Transmission	5-speed manual in unit with final drive
Gear ratios: 5th (0.757)	3.31:1
4th (0.971)	4.25:1
3rd (1.30)	5.68:1
2nd (1.94)	8.48:1
1st (2.92)	12.80:1
Final drive ratio	4.38:1

PRICE

List price, west coast	$11,482
Price as tested, west coast	$11,482

Price as tested includes standard equipment (power steering & brakes, self-leveling suspension, tinted glass, rear window heater, leather upholstery, air conditioning, AM/FM radio)

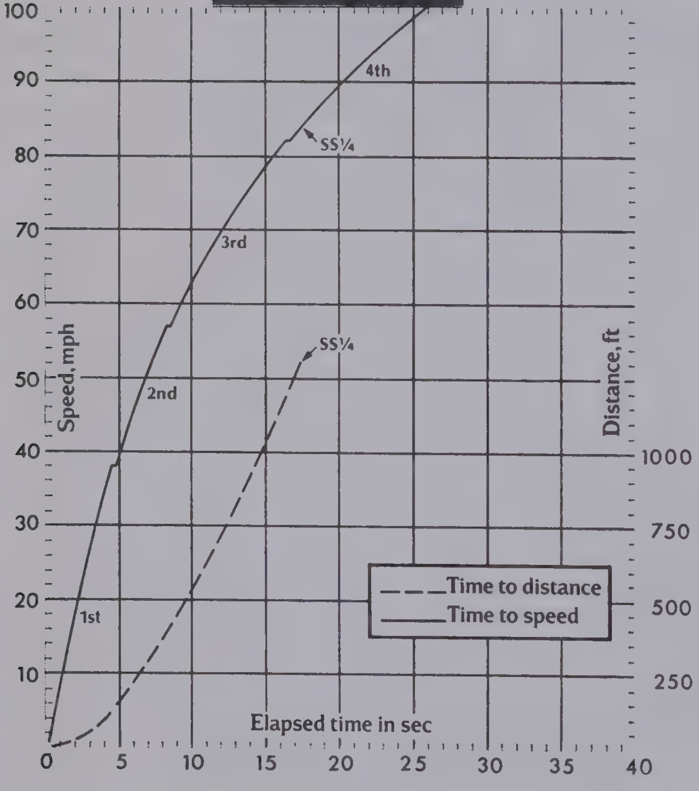
CALCULATED DATA

Lb/bhp (test weight)	20.2
Mph/1000 rpm (5th gear)	21.8
Engine revs/mi (60 mph)	2750
Piston travel, ft/mi	1350
R & T steering index	0.69
Brake swept area sq in/ton	210

ACCOMMODATION

Seating capacity, persons	2+2
Seat width, front/rear	2x19.0/52.0
Head room, front/rear	38.0/35.0
Seat back adjustment, degrees	30

ACCELERATION



ROAD TEST RESULTS

ACCELERATION

Time to distance, sec:	
0–100 ft	4.0
0–250 ft	6.5
0–500 ft	9.7
0–750 ft	12.5
0–1000 ft	14.9
0–1320 ft (¼ mi)	17.4
Speed at end of ¼ mi, mph	84.0
Time to speed, sec:	
0–30 mph	3.4
0–40 mph	5.1
0–50 mph	6.8
0–60 mph	9.3
0–70 mph	12.0
0–80 mph	15.5
0–100 mph	25.9
Passing exposure time, sec:	
To pass car going 50 mph	6.5

FUEL CONSUMPTION

Normal driving, mpg	15.9
Cruising range, mi	310

SPEEDS IN GEARS

5th gear (5950 rpm)	135
4th (6500)	115
3rd (6500)	82
2nd (6500)	57
1st (6500)	38

BRAKES

Panic stop from 80 mph:	
Max. deceleration rate, % g	84
Stopping distance, ft	300
Control	good
Pedal effort for 50%-g stop, lb	30
Fade test: percent increase in pedal effort to maintain 50%-g deceleration rate in 6 stops from 60 mph	nil
Parking: Hold 30% grade?	no
Overall brake rating	very good

HANDLING

Speed on 100-ft radius, mph	32.9
Lateral acceleration, % g	0.722

SPEEDOMETER ERROR

30 mph indicated is actually	27.0
40 mph	36.0
60 mph	55.0
70 mph	65.0
80 mph	74.0
90 mph	83.5
100 mph	92.0
Odometer, 10.0 mi	9.8

You might like to try Activity 2.2, pg. 113, *Doing Your Own Roadtest* or, Activity 2.7, pg. 117, *Monitoring A Car's Performance.*

ction B

22 One type of automobile speedometer is a small
ectric generator driven by a flexible cable run off
e drive shaft. The current produced increases with
e rate at which the generator is turned by the drive
aft. The speedometer needle indicates the current.
ntil the speedometer is calibrated it cannot indicate
tual speeds in miles per hour. Try answering the
uestions below.
) How would you calibrate the speedometer in a
r if the company had forgotten to do the job?
) If you replaced the 60 cm diameter rear wheels
ith 70 cm diameter wheels, what would your
tual speed be if the speedometer read 40 km
er hour?
) Would the speedometer read too high or too low
you loaded down the rear end of your car and had
e tire pressure too low.
) Does the operation of the speedometer itself
fect the motion of the car?
) How would you test to see if a bicycle speedo-
eter affects the speed of a bike?
Can you invent a speedometer that has no effect
n the motion of the vehicle that carried it?

23 The following world records are recorded for the
-mile race at 33-year intervals during the twentieth
entury:

an. 1, 1901 W. G. George (United Kingdom) 4 minutes 12.8 seconds set in 1886
lay 1, 1934 J. E. Lovelock (New Zealand) 4 minutes 07.6 seconds set in 1933
ept. 1, 1967 J. R. Ryan (United States) 3 minutes 51.1 seconds set in 1967

his record still stands at time of writing. Can you
se this data to predict the record in the year 2000.

2.24 World's 400-metre swimming records in minutes and seconds for men and women (numbers in parenthesis are ages):

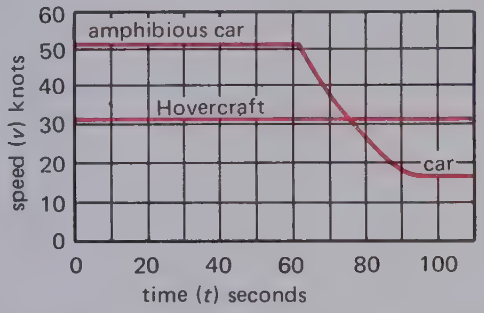
1926	4:57.0 5:53.2	Johnny Weissmuller (18) Gertrude Ederle (17)
1936	4:46.4 5:28.5	Syozo Makino (17) Helene Madison (18)
1946	4:46.4 5:00.1	(1936 record unbroken) R. Hveger (18)
1956	4:33.3 4:47.2	Hironoshin Furuhashi (23) Lorraine Crapp (18)
1966	4:11.1 4:38.0	Frank Weigand (23) Martha Randall (18)

By about how many metres would Martha Randall have beaten Johnny Weissmuller if they had raced each other? Could you predict the 1976 records for the 400-metre race by extrapolating the graphs of world's records versus dates up to the year 1976?

2.25 What problems arise when you attempt to measure the speed of light? Suggest an experiment to measure the speed of light.

* 2.26 Suppose you must measure the instantaneous speed of a bullet as it leaves the barrel of a rifle. Explain how you might do this.

2.27 A bullet leaves a rifle with a speed of 350 metres per second. If the length of the barrel is 0.75 metres, determine the acceleration of the bullet while in the gun. (Hint: Consider doing this problem in two steps.)



2.28 A hovercraft is able to travel over land and water at constant speed. An amphibious car is slowed down the instant it hits water.
The graph above describes the motions of each of these vehicles along a straight path toward a lake. The two vehicles are together at $t = 0$. Determine at what time the hovercraft hits the water's edge.

* 2.29 Two other equations that are useful in solving problems involving uniform acceleration are stated below. Use your earlier equations to derive them.

$$v_f^2 = v_i^2 + 2ad$$

$$d = v_f T - \tfrac{1}{2} a T^2$$

2.30 Why is this chapter entitled, *From the Complex to the Simple?*

Chapter 3 *Galileo Describes Motion*

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3.4 Galileo Chooses a Definition of Uniform Acceleration 38
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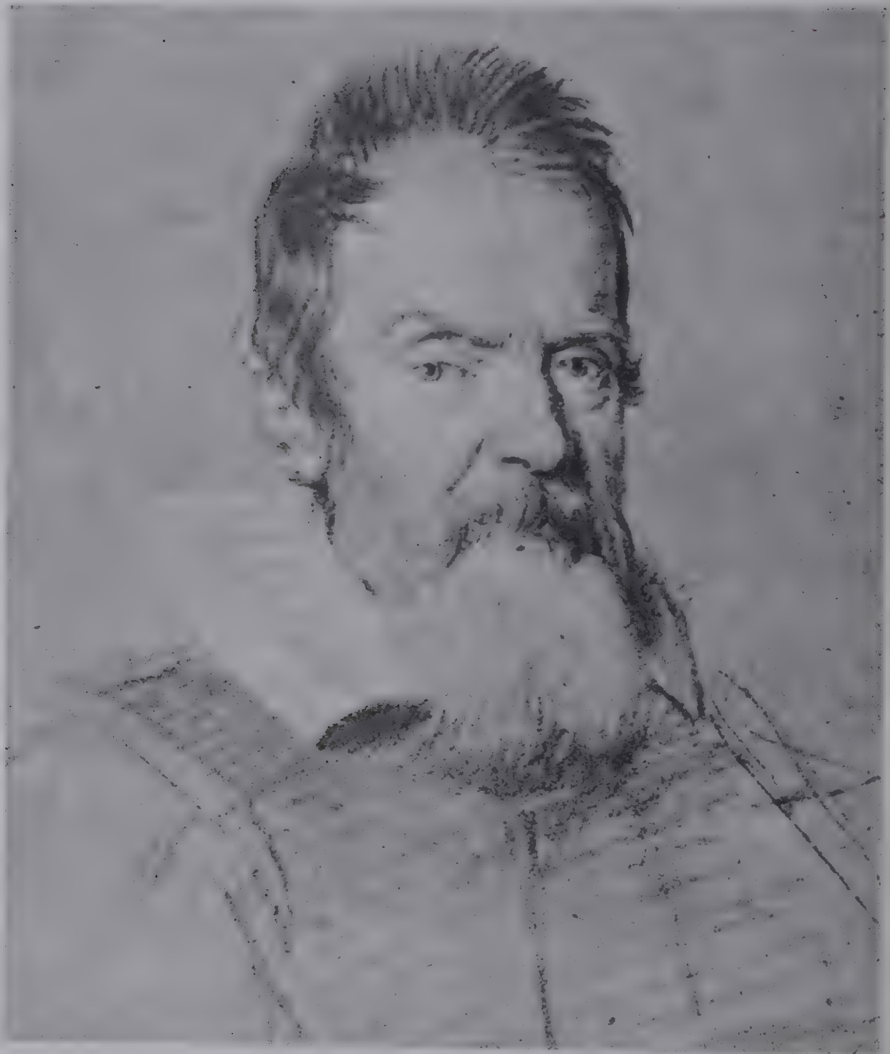


Galileo Describes Motion

Chapter Three

.1 Galileo Studies Free Fall

The discussion of motion in the last chapter was distinctly
ifferent from that in Chapter 1. There was no mention of natu-
al or violent motion, no attempt to show that objects move in
rder to fulfil some predetermined purpose. Rather, the emphasis
as placed on a few specific definitions, stated in words and
ymbolized by algebraic equations. Graphs made it possible to
epresent the motion of an entire trip with a few lines. Simple,
dealized motion was discussed rather than the complex motions
f the real world. Finally, there was no attention given to the
auses of these simple motions. We concentrated instead on
athematical descriptions of the characteristics of motion.

However, Chapter 1 did present an approach to science that
as typical of man's thinking for over two thousand years. Why
id a change occur? What conditions led to the change? How
ere old attitudes and prejudices challenged and overthrown?
hat kind of system replaced the old? The answers to these ques-
ions are important, not merely because they are a part of the
istory of science, but because in tracing this change, many of
he characteristics of modern science are brought into focus.

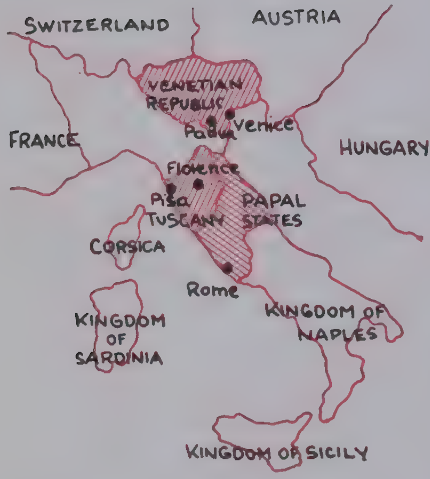
The world of 1500 A.D. had inherited from Thomas
quinas and his followers, two general attitudes necessary for the
rowth of science.

. From their intense study and discussion of the works of the
ncients, they had kept alive a faith in rational thought, reason,
nd logical analysis.

. Although sixteenth century men believed that nature could be
nderstood only through further interpretation of the writings of

earlier scholars, a continuing discussion of science and philosophy confirmed the belief that the natural world could be understood by man. This discussion encouraged an inquisitiveness about nature, without which scientific research could not have and cannot flourish.

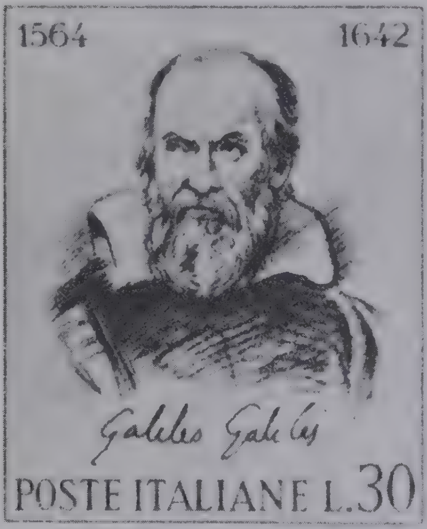
With the sixteenth century came that remarkably vigorous period of creativity and achievement known as the Renaissance. Literature, art, religion, and science all felt the impact of men who were developing new insights and techniques which were to fundamentally affect the traditions in each of these disciplines. To help understand the nature of the revolution in science, we shall focus on the work of one outstanding figure, Galileo Galilei. Galileo was born at Pisa in 1564, the year of Michelangelo's death and Shakespeare's birth. Rubens, Rembrandt, Milton, and Cervantes were all contemporaries of Galileo; Martin Luther and John Calvin had established the Reformation Movement; in England, Henry VIII had initiated the steps that were to remove the influence of Catholicism from the English monarchy. Nicholas Copernicus, a Pole, developed a theory that the sun, not the earth was the centre of the Universe.



Map of Italy at the time of Galileo.

Galileo began to reveal his scientific inventiveness early. For example, as a young medical student at the University of Pisa, he constructed a simple pendulum-type timing device for the accurate measurement of pulse rates.

Lured from medicine to physical science by reading Euclid and Archimedes, Galileo quickly became known for his unusual ability. At the age of 26, he was appointed Professor of Mathematics at Pisa. There he showed an independence of spirit unmellowed by tact or patience. Soon after his appointment, he began to challenge the opinions of his older colleagues, many of whom became his enemies. Indeed, he left Pisa before his term was completed, apparently forced out by financial difficulties and by his enraged opponents. Later, at Padua in the Republic of Venice, he began his work in astronomy. His support of the sun-centered theory of the universe eventually brought him additional enemies, but it also brought him immortal fame. You will read more about this in Unit 2.



Drawn back to his native province of Tuscany in 1610 by a generous offer of the Grand Duke, Galileo became the Court Mathematician and Philosopher, a title which he chose himself. From then until his death at 78 in 1642, he produced much of his excellent work. Despite illness, family troubles, occasional brushes with poverty, and quarrels with his enemies, he continued his research, teaching, and writing.

Galileo gave us a new mathematical orientation towards the natural world. His philosophy of science had its roots in the ancient Greek tradition of Pythagoras, Plato, and Archimedes, but it was in conflict with the qualitative approach characteristic of Aristotle. Unlike most of his predecessors, Galileo respected the test of truth provided by quantitative observation and experiment.

.2 Galileo's Two New Sciences

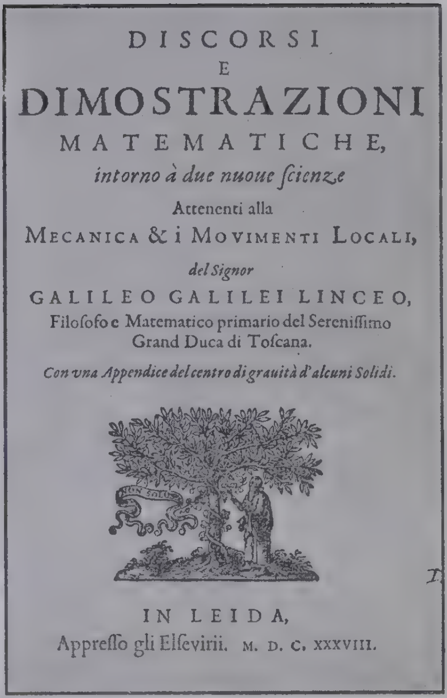
Galileo's early writings on mechanics (the study of the behaviour of matter under the influence of forces) were in the tradition of the standard medieval theories of physics. Although he was keenly aware of the shortcomings of those theories, his chief interest during his mature years was in astronomy. However, when his important astronomical work, *Dialogue on the Two Great World Systems* (1632) was condemned by the Roman Catholic Inquisition and he was forbidden to teach the "new" astronomy, Galileo decided to concentrate on mechanics. This work led to his book, *Discourses and Mathematical Demonstrations Concerning Two New Sciences Pertaining to Mechanics and Local Motion*, usually referred to as the *Two New Sciences.* The new approach to the science of motion described in the *Two New Sciences* signalled the beginning-of-the-end not only of the medieval theory of mechanics, but also of the entire Aristotelian cosmology.

Galileo was old, sick, and nearly blind at the time he wrote *Two New Sciences*, yet his style in it is spritely and delightful. He used the dialogue form to allow a lively conversation between three speakers: Simplicio, who represents the Aristotelian view: Salviati, who presents the new views of Galileo; and Sagredo, the uncommitted man of good will and open mind, eager to learn. To no one's surprise, Salviati leads his companions to Galileo's views. Let us listen to Galileo' s three speakers as they discuss the problem of free fall:

> *Salviati:* I greatly doubt that Aristotle ever tested by experiment whether it is true that two stones, one weighing ten times as much as the other, if allowed to fall at the same instant, from a height of, say, 100 cubits, would so differ in speed that when the heavier had reached the ground, the other would not have fallen more than 10 cubits. (A "cubit" is equivalent to about 51 centimetres.)
>
> *Simplicio:* His language would seem to indicate that he had tried the experiment, because he says: *We see the heavier;* now the word *see* shows that he had made the experiment.
>
> *Sagredo:* But, I, Simplicio, who have made the test can assure you that a cannon ball weighing one or two hundred pounds, or even more, will not reach the ground by as much as a span ahead of a musket ball weighing only half a pound, provided both are dropped from a height of 200 cubits.

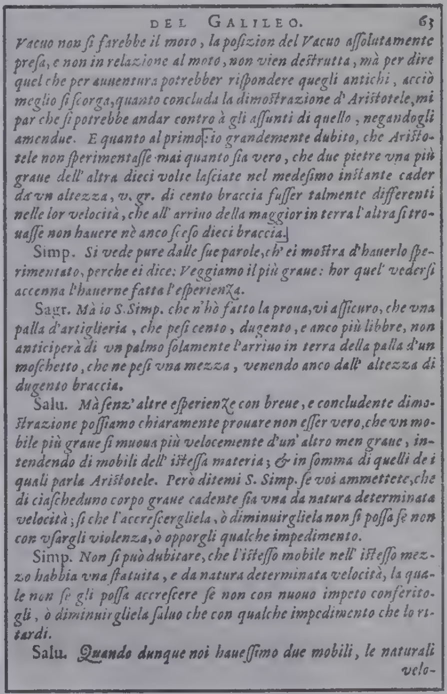
Here, perhaps, one might have expected to find a detailed report on an experiment done by Galileo or one of his colleagues. Instead, Galileo presents us with a "thought experiment"—an analysis of what would happen in an imaginary experiment, in which Galileo ironically uses Aristotle's own method of logical reasoning to attack Aristotle's theory of motion:

> *Salviati:* But, even without further experiment, it is possible to prove clearly, by means of a short and conclusive argument, that a heavier body does not move more rapidly than a lighter one provided both bodies are of the same material. . . . But tell me, Simplicio, whether you admit that each falling body acquires a definite speed fixed by nature, a velocity which cannot be increased or diminished except by the use of violence or resistance?



DISCORSI
E
DIMOSTRAZIONI
MATEMATICHE,
intorno à due nuoue ſcienze
Attenenti alla
MECANICA & i MOVIMENTI LOCALI,
del Signor
GALILEO GALILEI LINCEO,
Filoſofo e Matematico primario del Sereniſſimo
Grand Duca di Toſcana.
Con vna Appendice del centro di grauità d'alcuni Solidi.

IN LEIDA,
Appreſſo gli Elſevirii. M. D. C. XXXVIII.



DEL GALILEO. 63

Vacuo non ſi farebbe il moto, la poſizion del Vacuo aſſolutamente preſa, e non in relazione al moto, non vien deſtrutta, mà per dire quel che per auuentura potrebber riſpondere quegli antichi, acciò meglio ſi ſcorga, quanto concluda la dimoſtrazione d'Ariſtotele, mi par che ſi potrebbe andar contro à gli aſſunti di quello, negandogli amendue. E quanto al primo[io grandemente dubito, che Ariſtotele non ſperimentaſſe mai quanto ſia vero, che due pietre vna più graue dell' altra dieci volte laſciate nel medeſimo inſtante cader da vn altezza, v. gr. di cento braccia fuſſer talmente differenti nelle lor velocità, che all' arriuo della maggior in terra l'altra ſi trouaſſe non hauere nè anco ſceſo dieci braccia.]

Simp. *Si vede pure dalle ſue parole, ch' ei moſtra d'hauerlo ſperimentato, perche ei dice: Veggiamo il più graue: hor quel' vederſi accenna l'hauerne fatta l'eſperienza.*

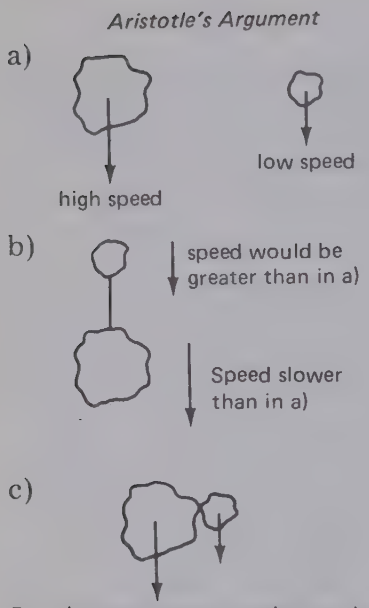
Sagr. *Mà io S. Simp. che n'hò fatto la proua, vi aſſicuro, che vna palla d'artiglieria, che peſi cento, dugento, e anco più libbre, non anticiperà di vn palmo ſolamente l'arriuo in terra della palla d'un moſchetto, che ne peſi vna mezza, venendo anco dall' altezza di dugento braccia.*

Salu. *Mà ſenz' altre eſperienze con breue, e concludente dimoſtrazione poſſiamo chiaramente prouare non eſſer vero, che vn mobile più graue ſi muoua più velocemente d'un' altro men graue, intendendo di mobili dell' iſteſſa materia; & in ſomma di quelli de i quali parla Ariſtotele. Però ditemi S. Simp. ſe voi ammettete, che di ciaſcheduno corpo graue cadente ſia vna da natura determinata velocità; ſi che l'accreſcergliela, ò diminuirgliela non ſi poſſa ſe non con vſargli violenza, ò opporgli qualche impedimento.*

Simp. *Non ſi può dubitare, che l'iſteſſo mobile nell' iſteſſo mezzo habbia vna ſtatuita, e da natura determinata velocità, la quale non ſe gli poſſa accreſcere ſe non con nuouo impeto conferitogli, ò diminuirgliela ſaluo che con qualche impedimento che lo ritardi.*

Salu. *Quando dunque noi haueſſimo due mobili, le naturali velo-*

A modern edition of *Two New Sciences,* translated by Crew and De Salvio is published by Dover Publications.



But the two stones together are heavier than either stone alone, and should fall faster than either body alone (according to Aristotle). Thus, Aristotle's argument is a contradiction.

Simplicio: There can be no doubt but that one and the same body moving in a single medium has a fixed velocity which is determined by nature and which cannot be increased except by the addition of impetus or diminished except by some resistance which retards it.

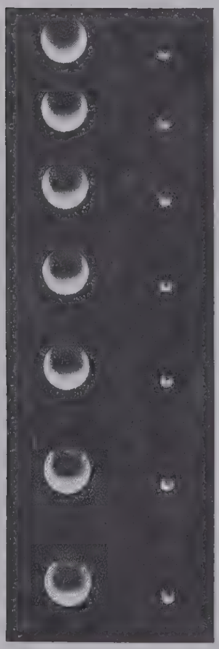
Salviati: If then we take two bodies whose natural speeds are different, it is clear that on uniting the two, the more rapid one will be partly retarded by the slower, and the slower will be somewhat hastened by the swifter. Do you not agree with me in this opinion?

Simplicio: You are unquestionably right.

Salviati: But if this is true, and if a large stone moves with a speed of, say, eight while a smaller moves with a speed of four, then when they are united, the system will move with a speed less than eight; but the two stones when tied together make a stone larger than that which before moved with a speed of eight. Hence the heavier body moves with less speed than the lighter; an effect which is contrary to your supposition. Thus you see how, from your assumption that the heavier body moves more rapidly than the lighter one, I infer that the heavier body moves more slowly.

Simplicio: I am all at sea. . . . This is, indeed, quite beyond my comprehension. . . .

As Simplicio retreats in confusion, Salviati presses forward with the argument, showing that it is self-contradictory to assume that an object would fall faster if its weight were increased by a small amount. Simplicio cannot refute Galileo's logic, but on the other hand his own eyes tell him that a heavy object *does* fall faster than a light object:



A photo of two freely falling balls of unequal weight. The balls were released simultaneously. The time-interval between images is 1/30 s.

Simplicio: Your discussion is really admirable; yet I do not find it easy to believe that a birdshot falls as swiftly as a cannon ball.

Salviati: Why not say a grain of sand as rapidly as a grindstone? But, Simplicio, I trust you will not follow the example of many others who divert the discussion from its main intent and fasten upon some statement of mine that lacks a hairsbreadth of the truth, and under this hair hide the fault of another that is as big as a ship's cable. Aristotle says that "an iron ball of one hundred pounds falling from a height of 100 cubits reaches the ground before a one-pound ball has fallen a single cubit." I say that they arrive at the same time. You find, on making the experiment, that the larger outstrips the smaller by two fingerbreadths. . . . Now you would not hide behind these two fingers the 99 cubits of Aristotle, nor would you mention my small error and at the same time pass over in silence his very large one.

This is a clear statement of an important principle; in careful observation of a common natural event the observer's attention may be distracted from a fundamental regularity unless he considers the possibility that small, separately explainable, variations will be associated with the event. Different bodies falling in air from the same height do *not* reach the ground at exactly the same time. However, the important point is not that the times of arrival are slightly *different*, but that they are *very nearly the same*! The failure of the bodies to arrive at exactly the same time is seen to be a minor matter which can be explained by a deeper understanding of motion in free fall. Galileo himself attributed the observed results to the resistance of the air. A few years after Galileo's death, the invention of the air pump allowed others to show that Galileo was right. When a feather and a heavy gold coin are dropped from the same height at the same time inside an evacuated container, they fall at the same rate and strike the bottom of the container at the same instant.

We might say that learning what to ignore has been almost as important in the growth of science as learning what to take into account. In this particular case, Galileo's explanation depended on his being able to imagine how an object would fall if there were no air resistance. This may be easy for us who know of vacuum pumps. But in Galileo' s time it was an explanation unlikely to be accepted because of the basic beliefs held by most educated people. For them, as for Aristotle, common sense said that air resistance is always present in nature. Thus, a feather and a coin could never fall at the same rate. Why should one talk about hypothetical motions in a vacuum, when a vacuum does not exist? Physics, said Aristotle and his followers, should describe the real world as we observe it, not some imaginary world which can never be found. Aristotle's physics had dominated Europe since the thirteenth century, not merely because of the authority of the Catholic Church, as is sometimes said, but also because many intelligent scientists were convinced that it offered the most rational method for describing natural phenomena. To overthrow such a firmly established doctrine required much more than writing reasonable arguments or simply dropping heavy and light objects from a tall building, as Galileo is supposed to have done in his legendary experiment from the Leaning Tower of Pisa. It demanded Galileo's unusual combination of mathematical talent, experimental skill, literary style, and tireless campaigning to defeat Aristotle's theories and to get on the path to modern physics.

* **Q1** Summarize Galileo's argument against Aristotle's claim that heavy bodies fall to earth faster than lighter bodies.

3.3 Why Study the Motion of Freely Falling Bodies?

By *Aristotelian cosmology* is meant the whole interlocking set of ideas about the structure of the physical universe and the behaviour of all the objects in it.

To attack the Aristotelian cosmology, Galileo gathered concepts, methods of calculation, and techniques of measurement in order to describe the motion of objects in a rigorous, mathematical form. Few details of his work were actually new, but together his findings provided the first coherent presentation of the science of motion. He realized that free-fall motion, now seemingly so trite, was the key to the understanding of all motions of all bodies.

Galileo also provides an example of a superb scientist. He was an investigator whose skill in discovery and eloquence in argument produced a deep and lasting impression on his listeners. His approach to the problems of motion will provide us with an opportunity for discussion of strategies of inquiry that are used in science. We shall see a new mode of scientific reasoning emerge, to become, eventually, an accepted pattern for scientific thought.

These are the reasons why we study in detail Galileo's attack on the problem of free fall. But perhaps Galileo himself should tell us why he studied motion:

> My purpose is to set forth a very new science dealing with a very ancient subject. There is, in nature, perhaps nothing older than motion, concerning which the books written by philosophers are neither few nor small; nevertheless, I have discovered some properties of it that are worth knowing and that have not hitherto been either observed or demonstrated. Some superficial observations have been made, as, for instance, that the natural motion of a heavy falling body is continuously accelerated; but to just what extent this acceleration occurs has not yet been announced. . . .
>
> Other facts, not few in number or less worth knowing I have succeeded in proving; and, what I consider more important, there have been opened up to this vast and most excellent science, of which my work is merely the beginning, ways and means by which other minds more acute than mine will explore its remote corners.

He was wrong in this: more than mere "superficial observations" had been made long before Galileo set to work. For example, Nicholas Oresme and others at the University of Paris had by 1330 discovered the same distance-time relationship for falling bodies that Galileo was to announce with a flourish in the *Two New Sciences.*

3.4 Galileo Chooses a Definition of Uniform Acceleration

In studying the following excerpts from the *Two New Sciences*, which deal directly with the motion of freely falling bodies, we must be alert to his overall plan. First, Galileo discusses the mathematics of a *possible*, simple type of motion, namely, motion with uniform acceleration. Then he *assumes* that this is the type of motion that a heavy body undergoes during free fall. Third, he deduces from this hypothesis some predictions that can be tested experimentally. Finally, he shows that these tests do indeed bear out the predictions.

It will help you to have this plan clearly in mind as you progress through the rest of this chapter. As you study each succeeding section, ask yourself whether Galileo is

- presenting a *definition*
- stating an *assumption*
- *deducing predictions* from his *hypothesis*
- experimentally *testing* the predictions.

In the first part of Galileo's presentation there is a thorough discussion of motion with uniform speed similar to the one in our Chapter 2. The second part concerns "uniformly accelerated motion":

> We pass now to . . . naturally accelerated motion, such as that generally experienced by heavy falling bodies. . . .
>
> And first of all it seems desirable to find and explain a definition best fitting natural phenomena. For anyone may invent an arbitrary type of motion and discuss its properties . . . we have decided to consider the phenomena of bodies falling with an acceleration *such as actually occurs in nature* and to make this definition of accelerated motion exhibit the essential features of observed accelerated motions.
>
> Finally, in the investigation of naturally accelerated motion we were led, by hand as it were, in following the habit and custom of nature herself, in all her various other processes, to employ only those means which are most common, simple, and easy. . . .
>
> When, therefore, I observe a stone initially at rest falling from an elevated position and continually acquiring new increments of speed, why should I not believe that such increases take place in a manner which is exceedingly simple and rather obvious to everybody? If now we examine the matter carefully we find no addition or increment more simple than that which repeats itself always in the same manner. This we readily understand when we consider the intimate relationship between time and motion; for just as uniformity of motion is defined by and conceived through equal times and equal spaces (thus we call a motion uniform when equal distances are traversed during equal time-intervals), so also we may, in a similar manner, through equal time-intervals, conceive additions of speed as taking place without complication. . . .

This is sometimes known as the *Rule of Parsimony*. unless you know otherwise, assume the simplest possible hypothesis to explain natural events.

Galileo is saying that just as we have defined uniform speed so that (to use our symbols, not his):

$$v = \frac{\Delta x}{\Delta t},$$

let us also define uniform acceleration so that:

$$a = \frac{\Delta v}{\Delta t},$$

Hence the definition of motion which we are about to discuss may be stated as follows:

A motion is said to be uniformly accelerated, when starting from rest, it acquires during equal time-intervals, equal increments of speed.

> *Sagredo:* Although I can offer no rational objection to this or indeed to any other definition devised by any author whosoever, since all definitions are arbitrary, I may nevertheless without defense be allowed to doubt whether such a definition as the foregoing, established in an abstract manner, corresponds to and describes that kind of accelerated motion which we meet in nature in the case of freely falling bodies.

Here Sagredo, the challenger, questions whether Galileo's rbitrary definition of acceleration actually corresponds to the ay real objects fall. Is acceleration, as defined, *useful* in desribing their change of motion? Sagredo tries to divert the onversation:

> From these considerations perhaps we can obtain an answer to a question that has been argued by philosophers, namely, what is the cause of the acceleration of the natural motion of heavy bodies. . . .

Salviati, the spokesman of Galileo, sternly turns away from is ancient concern for causes. It is premature, he declares, to sk about the cause of any motion until an accurate description f it exists:

Here Salviati refers to Aristotle's assumption that air propels an object moving through it. This point was discussed in chapter one.

> *Salviati:* The present does not seem to be the proper time to investigate the cause of the acceleration of natural motion concerning which various opinions have been expressed by philosophers, some explaining it by attraction to the centre, others by repulsion between the very small parts of the body, while still others attribute it to a certain stress in the surrounding medium which closes in behind the falling body and drives it from one of its positions to another. Now, all these fantasies, and others, too, ought to be examined; but it is not really worth while. At present it is the purpose of our Author merely to investigate and to demonstrate some of the properties of accelerated motion, whatever the cause of this acceleration may be.

Galileo has now introduced two distinct suggestions, which e must take up in turn.

1. "Uniform acceleration" means equal increases in speed Δv in equal times Δt.
2. Things actually fall that way.

et us first look more closely at Galileo's proposed definition.

Is this the only possible way of defining acceleration? Is it bviously right? Not at all! As Galileo goes on to admit, he once elieved that in uniform acceleration the speed increased in roportion to the distance travelled, Δx, rather than to the time t. In fact, both definitions had been discussed since early in the urteenth century, and both met Galileo's first command: ssume a *simple* relationship among the physical quantities ncerned. Furthermore, both definitions seem to match our mmonsense idea of acceleration. For example, when we say at a body is "accelerating", we seem to imply "the farther it es, the faster it goes", as well as "the longer it keeps moving, e faster it goes". And what, you might ask, is there to choose tween these two ways of putting it?

Acceleration could be defined either way. But which finition can be found useful in a description of nature? This is here experimentation is important. Galileo defined uniform celeration so that change of speed is proportional to elapsed me, and this definition led to fruitful consequences. Other scientists chose to define acceleration so that speed is proportional to distance traversed. Galileo's definition turned out to be the more useful so it was brought into the language of physics.

Q2 What was Galileo's definition for uniformly accelerated motion?

* Q3 Why did he choose this definition rather than some other one?

Q4 What is the difference between uniform speed and uniform acceleration?

3.5 Galileo Cannot Test His Hypothesis Directly

Galileo defined uniform acceleration so that it would match the way he believed freely falling objects behaved. The next task for Galileo was to show that the definition for uniform acceleration ($a = \Delta v/\Delta t$ = constant) was useful for describing observed facts.

This was not as easy as it seems. Suppose we drop a heavy object from several different heights—say, from windows on different floors of a building. In each case we observe the time of fall T and the speed v just before the object strikes the ground. Unfortunately, it would be very difficult to make direct *measurements* of the speed v just before striking the ground. Furthermore, the times of fall are smaller (less than 3 seconds even from the top of a 10-storey building) than Galileo could have measured accurately with the clocks available.

3.6 Looking for Logical Consequences of Galileo's Hypothesis

Galileo's inability to make direct measurements to test his hypothesis that $\Delta v/\Delta t$ is constant in free fall did not stop him from devising other means of testing it. He turned to mathematics to derive some other relationship that could be checked by measurement with the equipment available to him. Let us see how Galileo did this.

In Section 2.8, we showed that the distance travelled by a body in uniformly accelerated motion was equal to,

$$d = v_i T + \tfrac{1}{2} a T^2.$$

If the body is released from rest, then $v_i = 0$, and we have a simpler expression relating the distance fallen to the time taken.

$$d = \tfrac{1}{2} a T^2.$$

Galileo saw that this equation suggested a way of checking to see if objects really do fall to earth with constant acceleration. The reasoning is as follows.

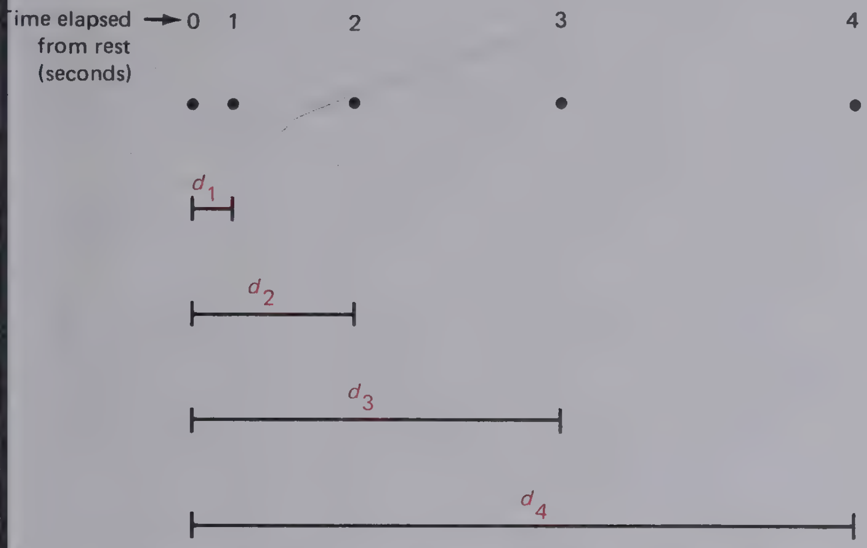
If the acceleration a is constant,

then $\tfrac{1}{2}a$ must also be constant.

But, $\tfrac{1}{2} a = \dfrac{d}{T^2}$

herefore, $\frac{d}{T^2}$ will be constant when the acceleration is constant.

Thus, a logical result of Galileo's original proposal for efining uniform acceleration can be expressed as follows. If an bject accelerates uniformly from rest, the ratio d/T^2 should be onstant. Conversely, any motion for which this ratio of d and ^{2} is found to be constant for different distances and the square f their corresponding time-intervals, will be a case of motion of niform acceleration, as defined by Galileo. The motion shown lustrates this point for a body uniformly accelerating along a raight line.



he body above, although not a falling body, is moving with uniform acceleration.

he ratio $\frac{\text{Distance travelled from rest}}{\text{Square of the time taken to travel that distance}}$ is constant.

Table 3.1

$\frac{d_1}{T_1^2} = 5 \frac{\text{mm}}{(1\text{s})^2}$ or $5 \frac{\text{mm}}{\text{s}^2}$

$\frac{d_2}{T_2^2} = 20 \frac{\text{mm}}{(2\text{s})^2}$ or $5 \frac{\text{mm}}{\text{s}^2}$

$\frac{d_3}{T_3^2} = 45 \frac{\text{mm}}{(3\text{s})^2}$ or $5 \frac{\text{mm}}{\text{s}^2}$

$\frac{d_4}{T_4^2} = 80 \frac{\text{mm}}{(4\text{s})^2}$ or $5 \frac{\text{mm}}{\text{s}^2}$

Fig. 3.1

It would seem that the problem now is simply one of easuring the time it takes a body to fall from rest through arious known distances and see if the ratio d/T^2 is constant for ll falls.

A practical problem arose for Galileo, however. Clocks, as e know them, had not been invented! There was no way that he short-intervals in which bodies fall small distances could be easured with sufficient accuracy to be of any value. So, instead f a direct test of his hypothesis, Galileo went one step further nd deduced an ingenious indirect test.

Q5 Why was the equation $d = 1/2\, aT^2$ more promising for Galileo than $a = \frac{\Delta v}{\Delta t}$ in testing his hypothesis?

Q6 What can be said about a body's motion when it moves such that:

i) the ratio $\frac{d}{T}$ is constant?

ii) the ratio $\frac{d}{T^2}$ is constant?

3.7 Galileo Turns to an Indirect Test

Realizing that it was still impossible to carry out direct quantitative tests with freely falling bodies, Galileo next proposed a related hypothesis which could be tested much more easily. According to Galileo, the truth of his new hypothesis would be established when he found that the inferences from it corresponded to and exactly agreed with the results of his experiment.

The new hypothesis is this: **if a freely falling body has an acceleration that is constant, then a perfectly round ball rolling down a perfectly smooth inclined plane will also have a constant, though smaller, acceleration.** Thus, Galileo claims that if d/T^2 is constant for a body falling freely from rest, this ratio will also be constant, although smaller, for a ball released from rest and rolling different distances down an inclined plane.

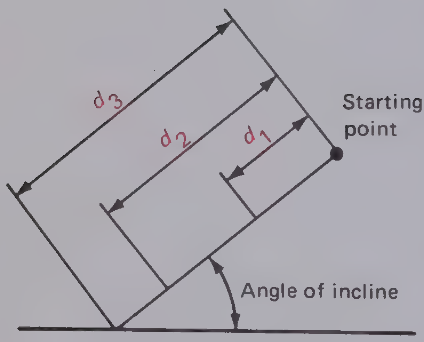
You can perform this experiment by following the procedure outlined in Experiment 3.1, pg. 119.

Here is how Salviati described Galileo's own experimental test:

Note the careful description of the experimental apparatus. Today an experimenter would add to his verbal description any detailed drawings, schematic layouts, or photographs needed to make it possible for any other competent scientist to duplicate the experiment.

> A piece of wooden moulding or scantling, about 12 cubits long, half a cubit wide, and three finger-breadths thick, was taken; on its edge was cut a channel a little more than one finger in breadth; having made this groove very straight, smooth, and polished, and having lined it with parchment, also as smooth and polished as possible, we rolled along it a hard, smooth, and very round, bronze ball. Having placed this board in a sloping position, by lifting one end some one or two cubits above the other, we rolled the ball, as I was just saying, along the channel, noting, in a manner presently to be described, the time required to make the descent. We repeated this experiment more than once in order to measure the time with an accuracy such that the deviation between two observations never exceeded one-tenth of a pulsebeat. Having performed this operation and having assured ourselves of its reliability, we now rolled the ball one-quarter of the length of the channel; and having measured the time of its descent, we found it precisely one-half of the former. Next we tried other distances, comparing the time for the whole length with that for the half, or with that for two-thirds, or three-fourths, or indeed for any fraction; in such experiments, repeated a full hundred times, we always found that the spaces traversed were to each other as the squares of the times, and this was true for all inclinations of the . . . channel along which we rolled the ball. . . .

Galileo has packed a great deal of information into these lines. He describes his procedures and apparatus clearly enough to allow other investigators to repeat the experiment for themselves if they wish; he gives an indication that consistent measurements can be made; and he restates the two experimental results which he believes support his free-fall hypothesis. Let us examine the results carefully.



First, he found that when a ball rolled down an incline at a fixed angle to the horizontal, the ratio of the distance covered to the square of the corresponding time was always the same. For example, if d_1, d_2, and d_3 represent distances from the starting point on the inclined plane, and T_1, T_2, and T_3 the corresponding times, then,

$$\frac{d_1}{(T_1)^2} = \frac{d_2}{(T_2)^2} = \frac{d_3}{(T_3)^2},$$

and in general (for a given angle of incline),

$$\frac{d}{T^2} = \text{constant.}$$

Galileo did not present his experimental data in detail, for that had not yet become the custom. However, his experiment has been repeated by others, and they have obtained results which parallel his. For example, one experimenter obtained the results shown in Table 3.2. But this is an experiment which you can perform yourself with the help of one or two other students.

Galileo's second experimental finding relates to what happens when the angle of inclination of the plane is changed. He found when the angle changed, the ratio d/T^2 also changed, although it was constant for any one angle. This was confirmed by repeating the experiment "a full hundred times" for each of many different angles. After finding that the ratio d/T^2 was constant for each angle of inclination for which measurements of T could be carried out conveniently, Galileo was willing to extrapolate.

He reasoned that the ratio d/T^2 is a constant even for larger angles where the motion of the ball is too fast for accurate measurements of T to be made. Further, he reasoned that if the ratio d/T^2 is constant when the angle of inclination is 90 degrees, then *d/T^2 is also a constant for a falling object.* Thus, by combining experimentation and reason, Galileo was able to make a convincing argument that for a falling object the ratio d/T^2 is a constant.

Q7 In Galileo's theory of falling bodies, the cause of the behaviour of the falling bodies is considered to be

a) the inherent simplicity of nature.

b) a power of "force" which is put into a body by the hand or other object that releases it.

c) an action exerted upon the body by a force which is present at all places about the earth, and which acts in the direction of the earth's centre.

d) the natural tendency of bodies to move horizontally with uniform motion, vertically with uniform acceleration.

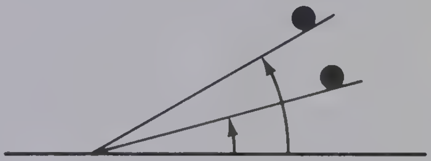
e) undetermined by Galileo's investigation and irrelevant, since the theory is merely concerned with giving a correct mathematical description of the behaviour of falling bodies.

3.8 How Valid was Galileo's Procedure?

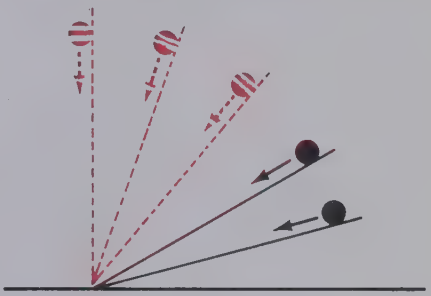
Some doubts arise concerning this whole process of reasoning and experimentation. First, was Galileo's measurement of time accurate enough to establish the constancy of d/T^2 even for the earlier case of a slowly rolling object? Galileo tries to reassure possible critics by providing a detailed description of his experimental arrangement (thereby inviting any skeptics to try it for themselves!):

Table 3.2 Results from an experiment of Thomas Settle in which the angle of inclination was 3°44′. (See *Science*, 133, 19-23, June 6, 1961).

Distance	Time (ml of water)	d/T^2
15 ft	90	.00185
13	84	.00183
10	72	.00192
7	62	.00182
5	52	.00185
3	40	.00187
1	23.5	.00182

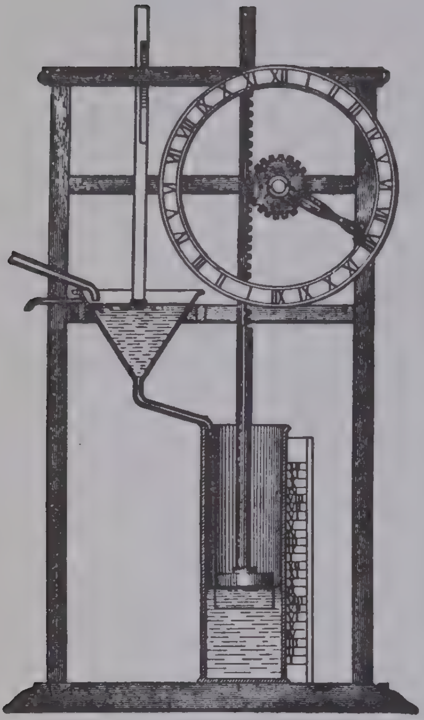


For each angle, the acceleration is found to be constant.



Spheres rolling down planes of increasingly steep inclination. At 90 degrees the inclined plane situation matches free fall. (Actually, the ball will start slipping long before the angle has become that large.)

> For the measurement of time, we employed a large vessel of water placed in an elevated position; to the bottom of this vessel was soldered a pipe of small diameter giving a thin jet of water, which we collected in a small cup during the time of each descent, whether for the whole length of the channel or for a part of its length; the water thus collected was weighed on a very accurate balance; the differences and ratios of these weights gave us the differences and ratios of the time-intervals and this with such accuracy that, although the operation was repeated many, many times, there was no appreciable discrepancy in the results.


An early waterclock.

The water clock described by Galileo was not invented by him. Indeed, there are references to water clocks in China as early as the sixth century B.C., and they were probably used in Babylonia and India even earlier. In Galileo's time, the water clock was the most accurate of the world's time-measuring instruments, and it remained so until shortly after his death when the work of Christian Huygens and others resulted in the pendulum clock. Although Galileo's own water clock was not the most precise available at the time, it was, nevertheless, good enough for a convincing verification that d/T^2 is constant.

Another reason for questioning Galileo's results is related to the large extrapolation involved. Galileo does not report what angles he used in his experiment. However, as you may have found out from doing a similar experiment, the angles must be kept rather small. Naturally, as the angle increases, the speed of the ball soon becomes so great that it is difficult to measure the times involved. The largest angle reported by Settle in his modern repetition of Galileo's experiment was only 6 degrees. It is unlikely that Galileo worked with much larger angles. This means that Galileo's extrapolation was a very large one, perhaps too large for a cautious person—or for one not already convinced of the truth of Galileo's hypothesis.

Extrapolation is the procedure of extending a graph line beyond its plotted limits.

Still another reason for questioning Galileo's results is the observation that, as the angle of incline is increased, there comes a point where the ball starts to slide as well as roll. This change in behaviour could mean that the same general law does not apply to both cases. Galileo does not answer this objection. It is surprising that he never repeated the experiment with blocks which would slide, rather than roll, down a smooth incline. If he had, he would have found that for both sliding and rolling the ratio d/T^2 is a constant although it is a different constant for two cases.

Q8 The main reasons why we might doubt the validity of Galileo's procedure are:
a) his measurement of time was not sufficiently accurate.
b) he used too large an angle of inclination.
c) it is not clear that his results apply to the case when the ball can slide as well as roll.
d) in Galileo's experiment the ball was actually rolling rather than sliding, and therefore his results cannot be extrapolated to the case of free fall.
e) d/T^2 would not be constant for a sliding object.

This picture painted in 1841 attempts to reconstruct an experiment Galileo is alleged to have made during his time as lecturer at Pisa. Off to the left and right are men of ill will: the blase Prince Giovanni de Medici (Galileo had shown a dredging-machine invented by the prince to be unusable) and Galileo's scientific opponents. These were leading men of the universities; they are shown here bending over a book of Aristotle, where it is written in black and white that bodies of unequal weight fall with different speeds. Galileo, the tallest figure left of centre in the picture, is surrounded by a group of students and followers. The painter, Bezzuoli, has placed the most important object in the middle of the picture: a little ball, which rolls downward in a sloping groove. With this Galileo provided the proof for a law which governs countless phenomena in nature: the behaviour of all bodies as they fall to the earth.

3.9 The Consequences of Galileo's Work on Motion

If you are skeptical of this conclusion try Activity 3.3, pg. 126.

As was pointed out at the end of the previous section, one cannot get the correct value for the acceleration of a body in free fall simply by extrapolating the results for larger and larger angles of inclination. In fact, Galileo did not even attempt to calculate a numerical value for the acceleration of freely falling bodies. Galileo's purpose could be well served without knowing the value of the acceleration for free fall; it was enough that he showed the acceleration down a ramp to be constant. This is the first consequence of Galileo's work.

Second, if spheres of different weights are allowed to roll down an inclined plane, they have the same acceleration. We do not know how much experimental evidence Galileo himself had for this conclusion. At any rate, later work confirmed his "thought experiment" on the rate of fall of bodies of different weights. The fact that bodies of different weights all fall at the same rate (aside from the understandable effects of air resistance) is a decisive refutation of Aristotle's theory of motion.

Third, Galileo developed a mathematical theory of accelerated motion from which other predictions about motion could be derived. We have already discussed some of these relationships in Chapter 2. All of the formulae relating distance, acceleration, time, and velocity that were derived there, are based on Galileo's definition of uniform acceleration as motion in which equal changes in velocity occur in equal time-intervals.

These consequences of Galileo's work, important as they are to modern physics, would scarcely have been enough to bring about a revolution in science by themselves. No sensible person in the seventeenth century would have given up his belief in the Aristotelian cosmology simply because its predictions had been refuted in the case of falling bodies. The significance of Galileo's work is that it prepared the way for the development of a new kind of physics, and indeed a new cosmology.

The more vexing scientific problem during Galileo's lifetime was not the motion of accelerated bodies, but the structure of the universe. For example, is the earth or the sun at the centre of the universe? Galileo supported the theory that the earth and other planets revolve around the sun. To accept such a theory meant, ultimately, to reject the Aristotelian cosmology; but in order to do this a physical theory of the motion of the earth would have to be developed. Galileo's theory of motion turned out to be just what was needed for this purpose, but only after it had been combined with further assumptions about the relation between forces and motion by the English scientist Isaac Newton. We shall return to the story of this revolution in science in Unit 2.

There is another significant aspect of Galileo's work on motion: it led to a new way of doing scientific research. The heart of this approach is the cycle, repeated as often as necessary: general observation→hypothesis→mathematical analysis→experimental test→modification of hypothesis as necessary in light of test, and so forth. But while the steps in the mathematical analysis are determined by "cold logic", this is not the case for the other elements of this process. Thus a variety of paths can lead to the hypothesis in the first place: an inspired hunch based on general knowledge of the experimental facts, a desire for simple and pleasing foundations, a change of a previous hypothesis that failed. Moreover, there are no general rules about how well the experimental data have to agree with the theoretical predictions. In some areas of science, a theory is expected to be accurate to better than one one-thousandth of one percent; in other areas, scientists would be delighted to find a theory that could make predictions with as little as 50 percent error.

The process of proposing and testing hypotheses, so skillfully demonstrated by Galileo in the seventeenth century, is widely used by scientists today. It is perhaps the most significant thing that distinguishes modern science from ancient and medieval science. The method is used not out of respect for Galileo as a towering figure in the history of science, but because it works so well so much of the time.

Galileo himself was aware of the value of both the results and the methods of his pioneering work. He concluded his treatment of accelerated motion by putting the following words into the mouths of the commentators in his book:

The "Academician" is the author of the treatise being discussed in the dialogues—that is, Galileo himself.

Sagredo: I think we may concede to our Academician, without flattery, his claim that in the principle laid down in this treatise he has established a new science dealing with a very old subject. Observing with what ease and clearness he deduces from a single principle the proofs of so many theorems, I wonder not a little how such a question escaped the attention of Archimedes, Appolonius, Euclid and so many other mathematicians and illustrious philosophers, especially since so many ponderous tomes have been devoted to the subject of motion.

Salviati: . . . we may say the door is now opened, for the first time, to a new method fraught with numerous and wonderful results which in future years will command the attention of other minds.

Sagredo: I really believe that . . . the principles which are set forth in this little treatise will, when taken up by speculative minds, lead to another more remarkable result; and it is to be believed that it will be so on account of the nobility of the subject, which is superior to any other in nature.

During this long and laborious day, I have enjoyed these simple theorems more than their proofs, many of which, for their complete comprehension, would require more than an hour each; this study, if you will be good enough to leave the book in my hands; is one which I mean to take up at my leisure after we have read the remaining portion which deals with the motion of projectiles; and this is agreeable to you we shall take up tomorrow.

Salviati: I shall not fail to be with you.

Q9 What conclusion can be reached from Galileo's work on free fall?

***Q10** In what sense could Galileo be regarded as the "father of modern science"?

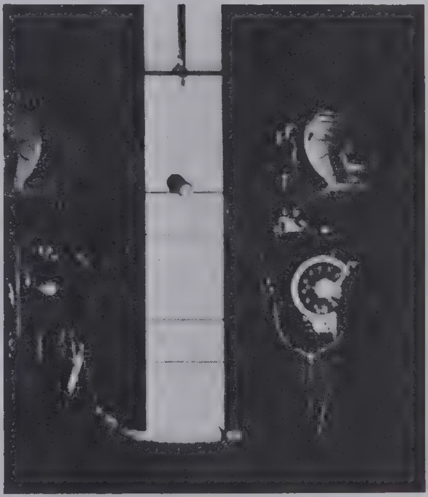
See Experiment 3.3, pg. 122.

.10 Measuring the Actual Acceleration of Freely Falling Bodies

With the instruments that are at our disposal today, there re many ways of measuring the actual acceleration of bodies in ree fall. Some experiments that can be done with inexpensive quipment and yield results within 10 percent of the accepted alue, are listed in your laboratory guide. One method is particuarly direct. We will discuss this method here because whether or ot you take actual measurements in the laboratory, a useful iece of information concerning uniformly accelerated motion merges.

Slow-motion pictures were taken of a ball falling freely past wo accurately measured intervals of distance. The pictures are eproduced in Fig. 3.2. By measuring the times T_1 and T_2 it akes for the ball to fall the distances d_1 and d_2, you can alculate the average speeds that the ball has during these two ntervals.

Fig. 3.2



$$v_{av_1} = \frac{d_1}{\underset{T_1}{\cancel{T_2}}} \text{ and } v_{av_2} = \frac{d_2}{T_2}.$$

ut in Galileo's definition of uniformly accelerated motion,

$$a = \frac{v_f - v_i}{T}$$

otice that v_f and v_i refer to instantaneous, not average speeds.

We can easily measure average speeds, but we need nstantaneous speeds for the equation. Let us see if there is a onnection between average and instantaneous speeds in a niformly accelerated motion.

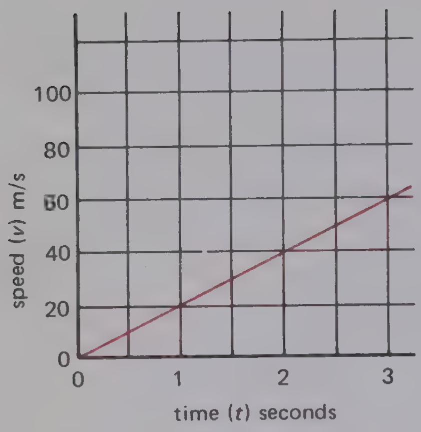


Fig. 3.3

The graph in Fig. 3.3 represents speeds that would actually occur at certain times in a uniformly accelerated motion. Recall from Chapter 2 that the average speed in a uniformly accelerated motion is just half the sum of the initial and final speeds.

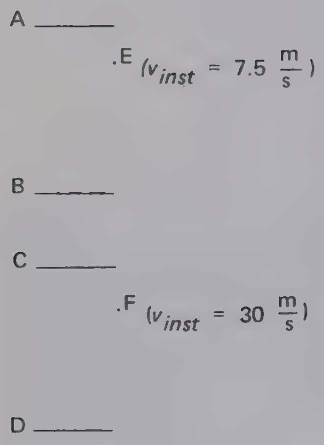
$$v_{av} = \frac{v_i + v_f}{2}$$

For the motion represented in Fig. 3.3, the average speed of the body between times one and three seconds would be:

$$\begin{aligned} v_{av} &= \frac{(20 + 60)}{2} \frac{\text{m}}{\text{s}} \\ &= \frac{80}{2} \frac{\text{m}}{\text{s}} \\ &= 40 \frac{\text{m}}{\text{s}} \end{aligned}$$

But this average speed is the actual speed that the body has at exactly two seconds. This time is exactly at the middle of the time-interval between the times one and three seconds. This result is perfectly general.

In any uniformly accelerated motion, the average speed actually occurs at the mid-time point of the interval over which the average speed is calculated.


A
.E $(v_{inst} = 7.5 \frac{m}{s})$
B
C
.F $(v_{inst} = 30 \frac{m}{s})$
D

Now, let us return to the falling ball. Although we can only measure average speed, we can easily determine the exact time that the instantaneous speed has the same value as the average speed. Suppose we find that it takes 0.4 seconds for the ball to fall from A to B, a distance of 3.0 metres. Then the average speed of the ball during this interval would be:

$$\begin{aligned} v_{av_1} &= \frac{d_1}{T_1} \\ &= \frac{3.0}{0.4} \frac{\text{m}}{\text{s}} \\ &= 7.5 \frac{\text{m}}{\text{s}}. \end{aligned}$$

The ball would actually have an instantaneous speed of 7.5 metres per second at the *mid-time* point of this interval, E, at 0.2 seconds after being at A.

Similarly,
if $d_2 = 3.0$ metres and $T_2 = 0.1$ seconds,
then,

$$\begin{aligned} v_{av_2} &= \frac{d_2}{T_2} \\ &= \frac{3.0}{0.1} \frac{\text{m}}{\text{s}} \\ &= 30 \frac{\text{m}}{\text{s}}. \end{aligned}$$

The ball would have this speed at a time 0.05 seconds after being at C.

The time taken for the ball to fall from B to C is 2.05

econds. We can calculate the actual acceleration of the ball as it ell from A to F as follows:

t E, (0.2 seconds after being at A),

$$v_{inst} = 7.5\ \frac{\text{m}}{\text{s}}.$$

(Call this $v_{initial}$.)

t F, (0.05 seconds after being at C)

r,

$$[(\underset{\substack{\text{time to fall}\\\text{from } A \text{ to } B}}{0.4} + \underset{\substack{\text{time to fall}\\\text{from } B \text{ to } C}}{2.05} + \underset{\substack{\text{time to fall}\\\text{from } C \text{ to } F}}{.05})\ \text{seconds after being at } A] = 2.5\ \text{s}$$

You may determine a_g using the procedure outlined in Film Loop Notes on pg. 128.

$$v_{inst} = 30\ \frac{\text{m}}{\text{s}}$$

(Call this v_{final}.)

herefore, the time taken to change speed from 7.5 metres per econd to 30 metres per second is equal to,

$$\begin{aligned} T &= (\text{time from } A \text{ to } F) - (\text{time from } A \text{ to } E)\\ T &= (2.5 - .2)\ \text{seconds}\\ &= 2.3\ \text{seconds.} \end{aligned}$$

hen,

$$\begin{aligned} a &= \frac{v_f - v_i}{T}\\ &= \frac{30 - 7.5}{2.3}\ \frac{\frac{\text{m}}{\text{s}}}{\text{s}}\\ &= 9.8\ \frac{\text{m}}{\text{s}^2}. \end{aligned}$$

The acceleration due to gravity is usually given the symbol a_g.

$a_g = 9.8\ \frac{\text{m}}{\text{s}^2}$ at the earth's surface

(or $32\ \frac{\text{ft}}{\text{s}^2}$).

This value, the free fall acceleration, or, the acceleration due o gravity, is the acceleration with which all bodies, of any mass hatsoever, fall to the earth's surface when they are dropped rom a point near the surface (assuming no air friction acts).

When bodies fall through the air, they accelerate at $9.8\ \frac{\text{m}}{\text{s}^2}$ nly for a certain period of time before the force of air friction auses changes in this acceleration. The length of time before this ccurs depends upon many factors too complicated to discuss ere. Fig. 3.4 graphs the way in which several different bodies hange their acceleration.

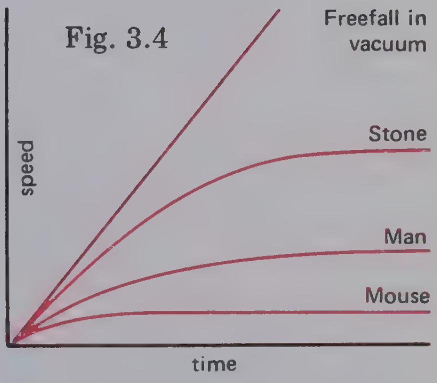


Fig. 3.4

Q11 What is meant by a freely falling body?

Q12 An acceleration of $9.8\ \frac{\text{m}}{\text{s}^2}$ for a freely falling body means that

a) it falls 9.8 metres each second?
b) its acceleration increases by $9.8\ \frac{\text{m}}{\text{s}^2}$ each second?
c) it travels $9.8\ \frac{\text{m}}{\text{s}}$ faster each second?
d) its speed averages $9.8\ \frac{\text{m}}{\text{s}}$ every second?
e) it falls 9.8 metres farther each second?

See Activity 3.5, pg. 127.

dditional problems using the freefall acceleration are on age 50.

Problems Chapter 3

Section A

When solving mathematical problems use the value 10 m/s^2 for the free fall acceleration rather than the correct value of 9.8 m/s^2. This will make your arithmetic simpler.

* 3.1 List the steps by which Galileo progressed from his first definition of uniformly accelerated motion to his final confirmation that this definition is useful in describing the motion of a freely falling body. Identify each step as a hypothesis, deduction, observation, or computation, etc. What limitations and idealizations appear in the argument?

* 3.2 The equations that we have derived so far in the text are listed below.

1. $v_{av} = \frac{d}{T}$
2. $a = \frac{v_f - v_i}{T}$
3. $d = v_i T + \frac{1}{2} a T^2$
4. $d = \left(\frac{v_i + v_f}{2}\right) . T$

The last two of these equations apply only to those cases where the acceleration is constant. Because these four equations are so useful, they are worth remembering.

Make up a *simple* problem to demonstrate the use of each equation. Also work out the solution just to be sure the problem can be solved.

3.3 A student observes that a ball bearing rolls from rest 50 cm down a ramp in the time it takes for 20 ml of water to flow through a funnel into a cylinder. What is the acceleration of the ball bearing?

3.4 A student on the planet Arret in another solar system dropped an object in order to determine the acceleration due to gravity. The following data are recorded (in local units):

Time (in surgs)	Distance (in welfs)
0.0	0.00
0.5	0.54
1.0	2.15
1.5	4.84
2.0	8.60
2.2	10.41
2.4	12.39
2.6	14.54
2.8	16.86
3.0	19.33

a) What is the acceleration due to gravity on the planet Arret, expressed in welfs/$surg^2$?

b) A visitor from Earth finds that one welf is equal to about 6.33 cm and that one surg is equivalent to 0.167 seconds.

What would these data tell us about Arret?

3.5 A rock dropped from the top of a cliff splashed into the water below 4.0 seconds after it was dropped.

a) How high was the cliff?

b) What was the speed of the stone just as it hit the water?

c) What was its average speed during its fall?

d) When would it have an instantaneous speed of 20 m/s?

3.6 A hammer falls from the top of a building 40 metres high. How long will it take to fall to the pavement below?

3.7 The following transcript is taken from the Apollo 15 mission to the moon. The experiment was performed July 31, 1972 by CDR David R. Scott. Duration of the experiment was 1.33 seconds. (The acceleration of freely falling objects on moon $= \frac{1}{6}$ that on earth.)

CC	Beautiful picture, Dave.
CDR-EVA	Well, in my left hand, I have a feather. In my right hand, a hammer. And I guess one of the reasons we got here today was because of a gentleman named Galileo a long time ago who made a rather significant discovery about falling objects in gravity fields. And we thought that—where would be a better place to confirm his findings than on the Moon? And so we thought we'd try it here for you. And the feather happens to be appropriately, a Falcon feather for our Falcon, and I'll drop the two of them here and hopefully, they'll hit the ground at the same time.
CDR-EVA	How about that?
CC	How about—
CDR-EVA	This proves that Mr. Galileo was correct in his findings.
CC	Superb.
CDR-EVA	Okay, let's see. What else do we have here?

a) Which of Galileo's findings was Commander Scott referring to?

b) From what height must Commander Scott have dropped the objects?

10c
AIR MAIL
FIRST MAN ON THE MOON

8 A freely falling object on a planet EREWHON celerates uniformly at 5.0 $\frac{m}{s^2}$.

How fast would an object be travelling after lling freely from rest for 1.0 second on this planet?

How far would it fall in 1.0 second if released om rest?

How far would it fall *during the second second* of drop?

9 Through what distance must a body fall from st in order to have an instantaneous speed of 30 $\frac{m}{s}$ that distance from the start?

10 A ball dropped from a window 45 metres above e ground accelerated uniformly until it hit the ound with a speed of 30 $\frac{m}{s}$.

How long did it take to hit the ground?

What was the average speed of the ball over the tire trip?

At what time would this speed actually occur?

At what distance below the window would this eed actually occur?

11 A cart sliding on a sloped air track starts from st and travels to the end of the 2-metre air track in seconds. What is the final speed of the cart?

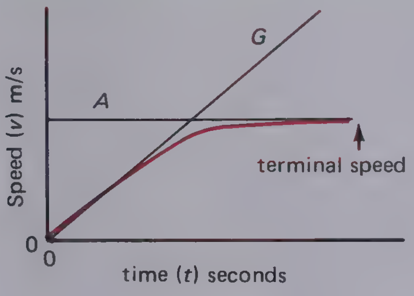
12 Show from a speed-time graph that the distances travelled in successive equal time-intervals by a dy accelerating uniformly from rest are in the ratio 1:3:5:7 . . . . (hint: Recall that the area under the e on a speed-time graph for a certain time-interval uals the distance travelled in that interval.)

13 All bodies, when falling through air, eventually ach *a terminal speed* and continue to fall with this nstant speed. The solid line in the speed-time graph ove shows the actual behaviour of a body falling rough the air, and the dotted lines represent the nd of motion that Aristotle (A) and Galileo (G) aimed represented the motion of a falling body.

Describe in your own words the kinematic operties represented by each line.

Based on this graph whose view, Aristotle's or lileo's, would you say is closer to reality?

s relationship may be investigated in Activity 3.6, pg. 127.



4 Galileo held that the proper language with which describe nature is mathematics. How is our understanding of natural phenomena aided by describing at we observe in mathematical terms?

Section B

* 3.15 As Director of Research in your class, you receive the following research proposals from physics students wishing to improve upon Galileo's free-fall experiment. Would you recommend support for any of them? If you reject a proposal, you should make it clear why you do so.

a) Historians believe that Galileo never dropped objects from the Leaning Tower of Pisa. Too bad! Such an experiment is more direct and more fun than inclined plane experiments, and of course, now that accurate stopwatches are available, it can be carried out much better than in Galileo's time. The experiment involves dropping, one by one, different sized spheres made of copper, steel, and glass from the top of the Leaning Tower and finding how long it takes each one to reach the ground. Knowing d (the height of the tower) and time of fall T, I will substitute in the equation $d = \frac{1}{2}aT^2$ to see if the acceleration a has the same value for each sphere.

b) A shotput will be dropped from the roof of a 4-storey building. As the shotput falls, it passes a window at each storey. At each window there will be a student who starts his stopwatch upon hearing a signal that the shot has been released, and stops the watch as the shot passes his window. Also, each student records the speed of the shot. From his own data, each student will compute the ratio v/T. All four students should obtain the same numerical value of the ratio.

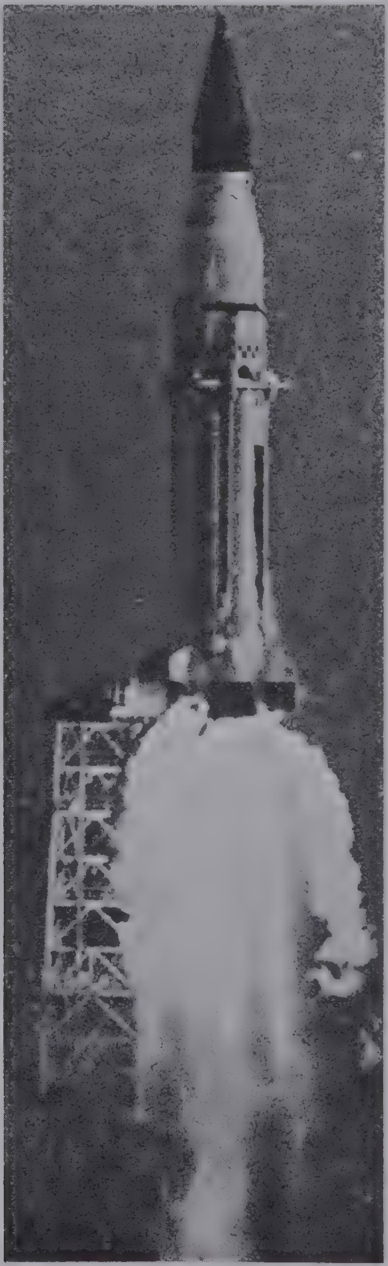
c) Galileo's inclined planes "dilute" motion all right, but the trouble is that there is no reason to suppose that a ball rolling down a board is behaving like a ball falling straight downward. A better way to accomplish this is to use light, fluffy, cotton balls. These will not drop as rapidly as metal spheres, and therefore it would be possible to measure the time of the fall T for different distances d. The ratio d/T^2 could be determined for different distances to see if it remained constant. The compactness of the cotton ball could then be changed to see if a different value was obtained for the ratio.

3.16 A ball is accidentally knocked down a hill and accelerates toward a pond at the bottom at 2.0 $\frac{m}{s}$2. A boy runs after the ball at a constant speed of 8 m/s reaching the top of the hill 2.0 seconds after the ball starts rolling down. It is 40 metres from the top of the hill to the pond. Will the boy get to the ball before it rolls in the pond? If so, where?

Chapter 4 *Kinematics: The Importance of Direction*

Section **Page**

4.1 Scalars and Vectors: The Importance of Direction 54
4.2 Graphical Analysis 56
4.3 Algebraic Analysis Revisited 60
4.4 Motion on a Surface 62



Kinematics: The Importance of Direction

Chapter Four

There are three fundamental questions which arise as we pursue the topic of "motion". They are:

1. How does an object move? (*pattern*)
2. Why does an object move? (*cause*)
3. When does an object move? (*relativity*)

The question: Why does an object move? is a topic of discussion in Chapter 5.

The early scientists attempted to find a pattern for various movements by collecting data and searching for a geometrical or algebraic structure which could be made to fit those data. They also spent time theorizing, in philosophical terms, about various likely causes of movement. The significance of the third question, when does an object move?, can be better grasped by considering the following examples.

1. If you are sitting on a ship and observe that the distance between the ship and the shore is increasing should you say that the ship is moving away from the shore or that the shore is moving away from the ship?
2. If you observe a book on a table should you describe it as being at rest even though you know that the table is sitting on the spinning earth?
3. Should our daily observation of the westward passage of the sun be described in terms of the sun circling the earth or in terms of the earth's motion?
4. How do you describe the motion of a passenger in a moving train when he sits in his seat, walks toward the engine, or walks toward the caboose?

"Look how quickly the shore is speeding away from me."

Discussion of questions of this type soon lead to the conclusion that in order to describe motion one must decide on a specific point of reference. Then the motion can be described relative to that point. Thus a book sitting on a table may be described by an observer sitting in the room as being at rest although from the viewpoint of a person walking across the room it is considered to be in motion. Thus *an object is described* as

This definition raises another question. Is there any point in the universe that is absolutely at rest?

being in motion with respect to an observer, *if a line joining the object to the observer changes in either length or direction.* This is a useful definition which suggests that motion be described from a *convenient* viewing point or frame of reference. In our previous description we discussed motion from the point of view of an observer fixed on the earth. We often choose this point of view, not because it is more correct than any other, but because it is more convenient.

See Film Loop Notes 4.1, *A Matter of Relative Motion,* pg. 130.

In our discussion of simple motion in Chapter 2 we considered the motion of an object travelling in one direction along a line. We saw that from the viewpoint of an observer fixed on the earth its motion could be simply described by distance-time and speed-time graphs and by algebraic equations. These mathematical methods are adequate to extend our description of moving objects to include motion in either direction on a line. We can also develop methods of describing motion that is not restricted to a line but is permitted to move in any direction on a surface. The mathematics of vector algebra was developed in consideration of the importance of direction.

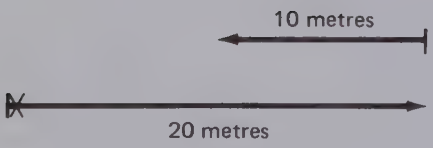
Q1 Describe the motion of each of the following objects relative to a car moving north at 100 kilometres per hour with respect to the road.

a) A hitchhiker standing beside the road.
b) Another car heading north at 100 kilometres per hour.
c) An airplane overhead flying south at 300 kilometres per hour.
d) A car heading north at 101 kilometres per hour.

4.1 Scalars and Vectors: The Importance of Direction

The car pictured in the margin is approaching a cliff at a constant speed of 10 metres per second. The driver is no doubt reluctant to continue providing an example of uniform motion! He will probably slow down, stop, admire the view, and then turn back along the road. Let us consider how we can describe these motions. We shall find that some of the terminology developed to describe simple motion in one direction is somewhat inadequate and that we shall have to redefine some of the terms we have used.

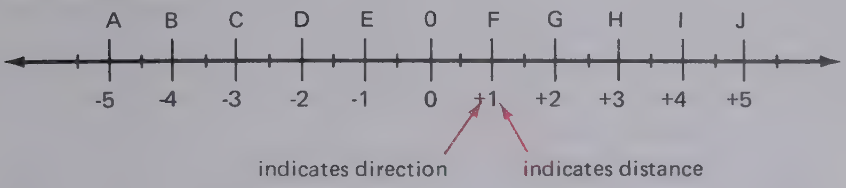
Let us start with an analysis of the following motion. Consider an object which starts at a point x, and proceeds for 20 metres to the right. The object then stops and proceeds from this point 10 metres to the left where it stops again. This motion is shown in the marginal sketch. The following questions arise.



1. Suppose the object had finished at a point 10 metres to the left of the starting point. How would you distinguish this from a position 10 metres to the right of the starting position?
2. What is the total distance the object travelled?
3. What is the net change in the position of the object as a result of its motions?

Your consideration of the first question probably resulted in your deciding on a method of distinguishing between a position to the right and a position to the left of the starting point. One simple method of doing this is to indicate positions to the right by a positive sign in front of the distance from the starting point, and positions to the left by means of a negative sign. This is similar to setting up a number line in mathematics. The number line below indicates the positions of several points by using both a + or − sign to distinguish between positions to the right and left, and a number which indicates the distance from the origin.

The origin is an arbitrary point chosen for convenience. It often is chosen to represent a starting point.



The position of H is + 3. (3 units of distance to the right of the origin.) Position of A is − 5. (5 units of distance to the left of the origin.)

Distance between Toronto and Montreal is 560 kilometres

Displacement Montreal, Toronto is 560 kilometres S.W.

Displacement Toronto, Montreal is 560 kilometres N.E.

In considering questions (2) and (3), you probably realized that there is a difference between the total distance the object travels, which is 30 metres, and the actual change in the objects position. The net effect of this 30 metre journey is to move the object to a position 10 metres to the right of the starting point, (+ 10m). Scientists use the terms **distance** and **displacement** to distinguish between these two different ideas.

The *distance* the object travelled is 30 metres.

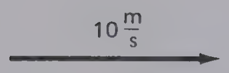
The *displacement* of the object from the starting point is 10 metres to the right (+ 10m). When the term displacement is used the distance and direction of the final position relative to the initial position are given.

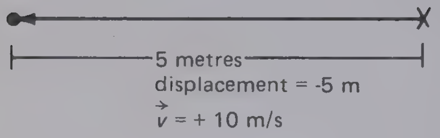
We can further illustrate the concept of displacement by considering again the number line illustrated previously. The displacement of position H measured relative to position A is $+8$. ($\overrightarrow{AH} = +8$) The displacement of E from I is -5. ($\overrightarrow{IE} = -5$). And the displacement of H from O is + 3. ($\overrightarrow{OH} = +3$). Note that when displacement is measured from the origin the displacement is the same as the position indicated on the number line.

The arrow over the IE indicates that displacement is a vector quantity (discussed on the next page).

Now consider three more questions. Suppose that the total time for the trip described in our previous example in which an object travelled 20 m to the right and then 10 m to the left is 6 seconds.

1. What is the average speed of the object?
2. How can we distinguish between an object moving at 20 metres per second to the right of a starting point, and an object moving at 20 metres per second to the left?
3. Should we calculate speed by dividing the distance travelled by the time taken or by dividing the displacement by the time taken?





It is from consideration of questions such as these that scientists have found it necessary to distinguish between the terms *speed* and *velocity*. **Speed** *is defined as the distance travelled per unit time.* In the example above, the speed is 30 metres in 6 seconds or 5 metres per second. **Velocity** *is defined as the rate of change of position.* It is calculated by dividing the displacement by the total time. In the example above the velocity would be 10 metres to the right in 6 seconds or 1.67 metres per second to the right. When stating a velocity both the magnitude and direction of travel are given. Thus if we use the previously suggested sign convention for displacement, we would indicate velocities to the right with a positive sign and those to left with a negative sign. For example an object having a displacement of —5 metres and a velocity of + 10 metres per second would be positioned 5 metres to the left of a reference point and travelling at 10 metres per second toward the right.

A scalar quantity has magnitude.

A vector quantity has both magnitude and direction.

Speed, distance between per unit time, is a *scalar quantity.*

Velocity, displacement (distance from . . . to) per unit time, is a *vector quantity.*

Quantities represented not only by magnitude but also by direction are called **vector quantities.** Displacement and velocity are *vector quantities.* Quantities which can be represented by magnitude only are called **scalar quantities.** Speed, distance, and mass are scalar quantities. Acceleration is considered a vector quantity and hence the direction of an acceleration should be stated. The same sign convention for displacement and velocity can be used for acceleration and any of the other vector quantities we shall discuss in this course. In fact this type of sign convention is not only useful in describing motion to the right or left but applies to any motion along a line. If the line runs north-south, east-west, or up-down the same sign convention may be used by simply designating one direction as positive and the opposite direction as negative.

Sign Convention for Motion Across Page

	Direction	
	right	left
Displacement	+	—
Velocity	+	—
Acceleration	+	—

Q2 Distinguish between the following pairs of expressions. Distance and Displacement; Velocity and Speed; Scalar and Vector Quantities.

Q3 If a giraffe travels 25 metres to the left and then returns a distance of 10 metres to the right in 5 seconds, what is its displacement and distance travelled from the starting point?

Determine the average speed and velocity of the giraffe.

Q4 If a runner completes a circuit of a 300-metre track in 40 seconds, determine his average speed and average velocity.

Also see problems 4.1 to 4. 4 on page 65.

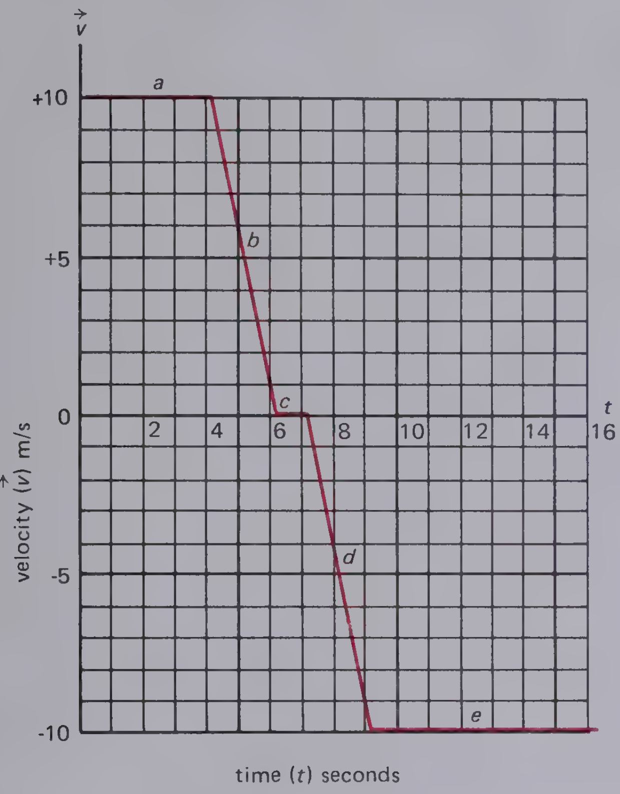
The importance of direction is considered in Experiment 4.1, pg. 129.

4.2 Graphical Analysis

Let us now use our established sign convention to describe graphically what happens when the car which was heading toward the cliff slows down, stops, and returns toward the starting point. The graph in Fig. 4.1 shows the following stages in the trip. a) The car travels to the right at 10 metres per second for 4 seconds. b) The car then slows down uniformly to a stop in

2 seconds. c) The car remains stopped for one second. d) The car accelerates uniformly for 2 seconds to a velocity of 10 metres per second to the left. e) Then the car travels to the left at a constant velocity of 10 metres per second.

Fig. 4.1



Note the following:

1. The direction of motion is simply shown on the graph. Velocities to the right are above the time axis and those to the left, below.

2. The slope of the line in the velocity-time graph in sections *b* and *d* is the same value, and negative. This indicates that the object was accelerating to the left at the same rate in both cases. Because the object at section *b* is moving to the right the acceleration to the left causes the object to slow down. At *d*, however since the motion and the acceleration are in the same direction, the speed increases.

Effect of Acceleration on Speed.

Velocity Direction	+ —	+ —
Acceleration Direction	+ —	— +
Speed	Increases	Decreases

3. The numerical value of the area under the line on a speed-time graph equals the numerical value of the distance travelled during the time interval. If we consider areas above the horizontal time axis to correspond to positive displacements and those below to correspond to negative displacements, we can use this graph to determine the position of the car. The graphs in Fig. 4.2 will assist us in this analysis.

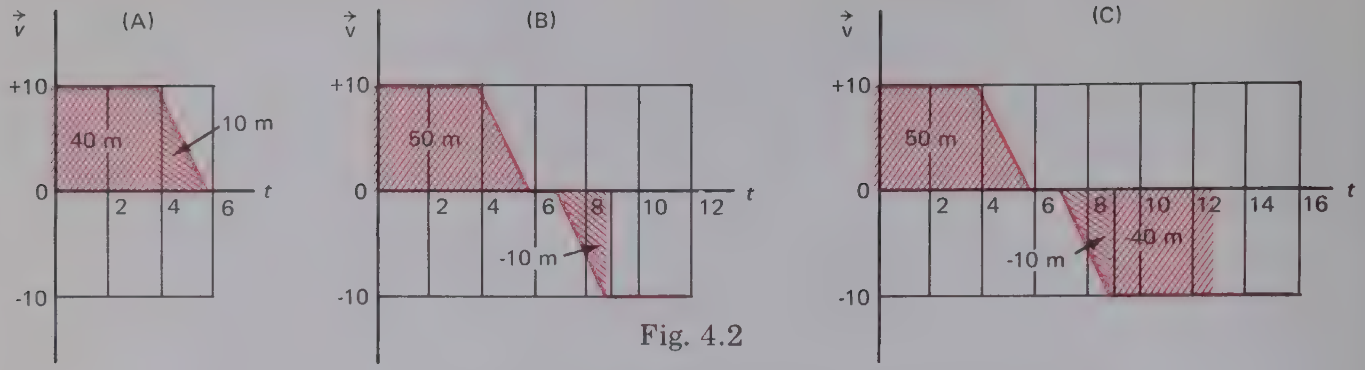


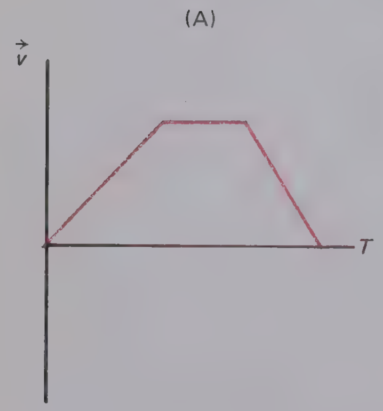
Fig. 4.2

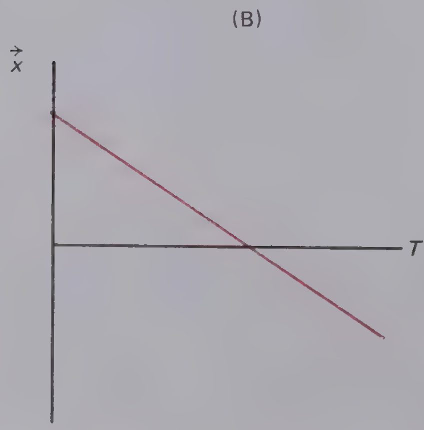
The unit metres represents the quantity measured in determining the area under this graph, since on the graph speed is measured in metres per second and time in seconds.

Fig. 4.2a considers the first 6 seconds of motion.
Area under velocity-time graph is area of rectangle + area of triangle = (+40 m) + (+10m) = + 50 m.
Displacement of car is therefore + 50m after 6 seconds.
Fig. 4.2b considers 9 seconds of motion
Area under velocity-time graph is area under horizontal section + area of triangle = (+ 50 m) + (—10 m) = + 40 m.
The displacement of the car is + 40 m after 9 seconds.
Fig. 4.2c considers 13 seconds of motion
Area under velocity-time graph for first 13 seconds = (+ 50 m) + (—10 m) + (—40 m) = 0 m.
Total displacement of car is therefore 0 m after 13 seconds. That is, the car has returned to its starting point.

A complete set of motion graphs for this object is drawn in Fig. 4.3. Look at these graphs. Do you see how they indicate both magnitude and direction in their description of motion? The arrow over the variables $\vec{x}$, $\vec{v}$, $\vec{a}$ indicates that they are vector quantities.

Q5 Describe in a few sentences the motions depicted by graphs *A*, *B*, and *C*.
Q6 Sketch the corresponding velocity-time graph for graphs *B* and *C*.
Q7 Sketch the corresponding position-time graph for graph *A*.
Also see problems 4.5 and 4.6 on page 66.





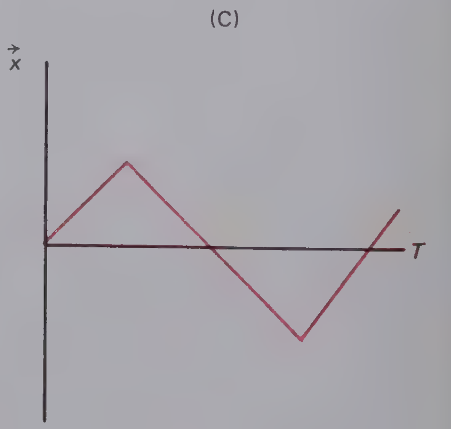
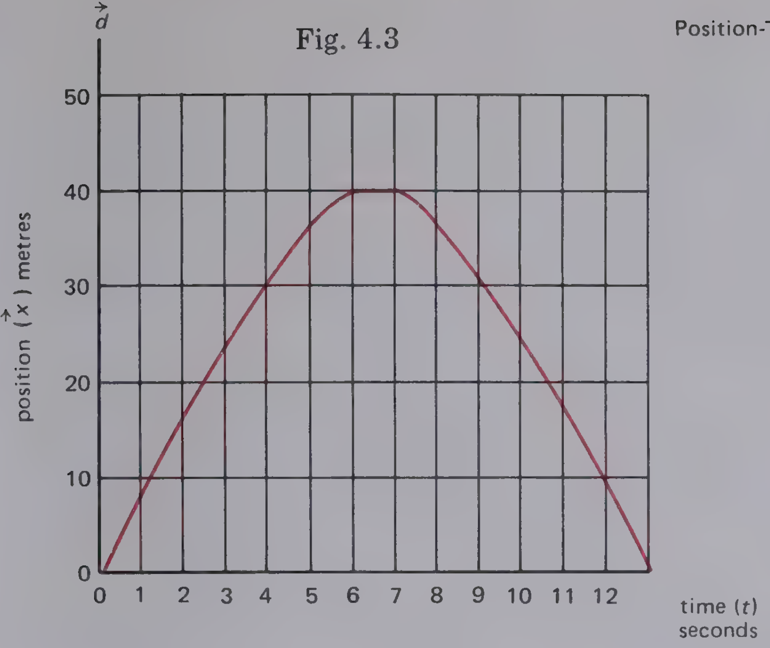


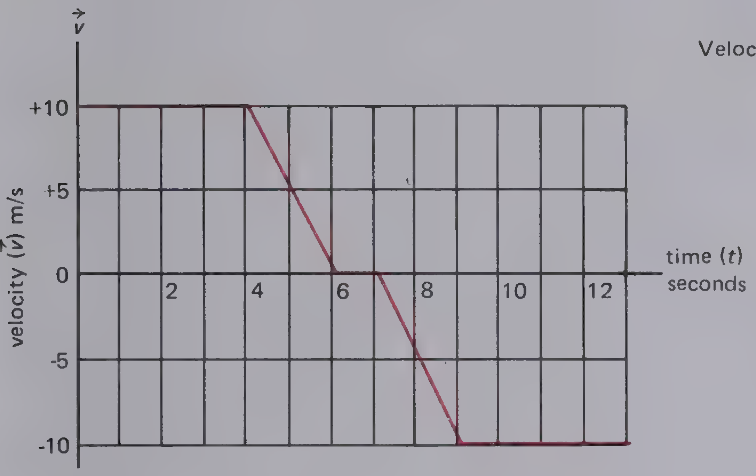
Fig. 4.3

Position-Time Graph

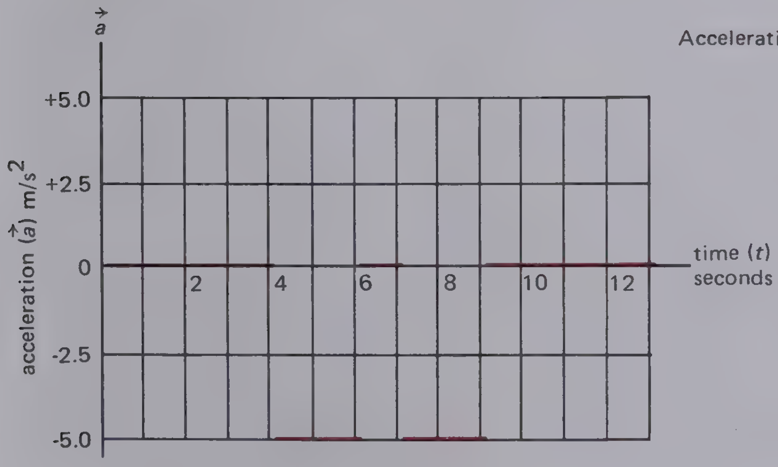


The information on any one of the graphs describes the motion. By analyzing any one graph you can derive the other two.

Velocity-Time Graph



Acceleration-Time Graph



4.3 Algebraic Analysis Revisited

The equations derived earlier (in Chapter 2) to describe motion in one direction in a straight line may be used to describe motions in both directions on a line if we consider the symbols to represent vectors instead of scalars. Let $\vec{d}$ = displacement from the starting point, $\vec{v}$ = velocity, and $\vec{a}$ = acceleration. With this change, the algebraic equations are able to describe this slightly more complicated form of motion. This can be illustrated by the following problem.

Use the equation $\vec{d} = \vec{v}_i T + \frac{1}{2}\vec{a}T^2$ to sketch the position-time graph for an object initially travelling at 20 metres per second to the right and accelerating uniformly at 5 metres per second per second to the left. Then use the equation $\vec{v}_f = \vec{v}_i + \vec{a}T$ to sketch the velocity-time graph.

Note that we have used the symbols + and — to indicate both directions and operations in our equations. In these first two examples we have set the signs of direction in color. This will not be continued in later examples.

As before we have used the sign convention of indicating vectors to the right as positive and those to the left as negative. Solving these two equations for T = 3 seconds, gives the following values for position and velocity.

$$\begin{aligned}
\vec{d} &= \vec{v}_i T + \tfrac{1}{2}\vec{a}T^2 \\
&= (+20\ \text{m/s} \times 3\ \text{s}) + \tfrac{1}{2}(-5\ \text{m/s}^2)(3\ \text{s})^2 \\
&= 60\ \text{m} - \tfrac{1}{2}(5\ \text{m/s}^2)(9\ \text{s}^2) \\
&= 60\ \text{m} - 22.5\ \text{m} \\
&= +37.5\ \text{m}
\end{aligned}$$

The position of the object after 3 seconds is 37.5 metres to the right of the starting point.

$$\begin{aligned}
\vec{v}_f &= \vec{v}_i + \vec{a}T \\
&= +20\ \text{m/s} + (-5\ \text{m/s}^2 \times 3\ \text{s}) \\
&= 20\ \text{m/s} - 15\ \text{m/s} \\
&= +5\ \text{m/s}
\end{aligned}$$

The velocity of the object after 3 seconds is 5 metres per second to the right.

The values of the position and velocity at one second intervals for this motion have been calculated and recorded in the charts and graphs in Fig. 4.4. Verify some of the values in the chart and examine the two graphs.

Note that the numerical values of the slope of the position-time graph and the velocity are equal, and that the numerical values of the area under the curve in the velocity-time graph and the displacement from the starting point are also equal.

To complete our set of motion graphs for this example we have included the acceleration-time graph for the object in Fig. 4.4.

Equation of Curve

Given Values and Calculations

Graphs

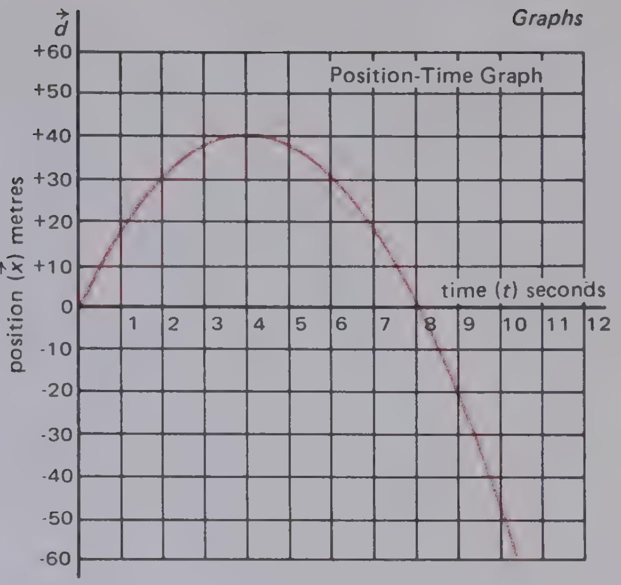
If $\vec{v}_i = +20$ m/s

$\vec{a} = -5$ m/s^2

$\vec{d} = \vec{v}_i T + \frac{1}{2}\vec{a}T^2$

$\vec{d} = +20T - 2.5T^2$

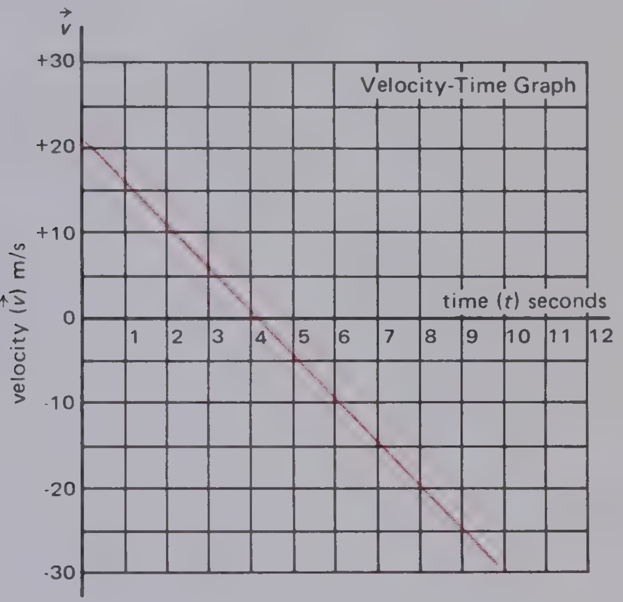
Time elapsed from start T (s)	Displacement from rest $\vec{d}$ (m)
0	0
1	+17.5
2	+30.0
3	+37.5
4	+40.0
5	+37.5
6	+30.0
7	+17.5
8	0
9	-22.5
10	-50.0



$\vec{v}_f = \vec{v}_i + \vec{a}T$

$\vec{v}_f = +20 - 5T$

time (t) seconds	velocity ($\vec{v}$) m/s
0	+2
1	+15
2	+10
3	+5
4	0
5	-5
6	-10
7	-15
8	-20
9	-25
10	-30



$\vec{a} = -5$ m/s^2

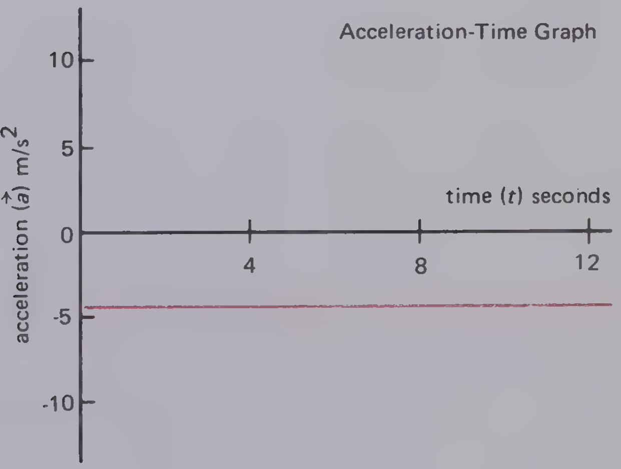


Fig. 4.4

The acceleration is a constant $-5\ \mathrm{m/s^2}$ which means that the object is accelerating to the left at a constant rate of 5 metres per second per second. Note that the acceleration is equal to the slope of the velocity-time graph.

These three graphs and equations describe the motion of an object which is initially moving in one direction while experiencing a constant acceleration in the opposite direction. The object slows down, stops, reverses direction, and then speeds up in the direction opposite to its initial motion. This is the situation that occurs when an object is thrown straight up into the air. You might like to re-examine these graphs considering this real situation. (In the real case on earth the acceleration is $9.8\ \mathrm{m/s^2}$.)

Q8 An object is travelling to the left at 8 metres per second and in accelerating uniformly to the right at 2 metres per second per second. Describe its motion by means of:

a) equations to state its position and velocity at any time.
b) charts indicating its position and velocity at one second intervals for the first 10 seconds of motion.
c) graphs showing the position and velocity during the first 10-second interval.

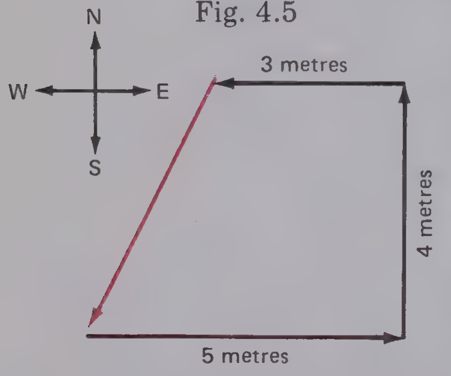
Q9 An object is travelling to the left with a velocity of 20 metres per second and accelerating to the right at 5 metres per second per second. Use an algebraic expression to calculate the position and velocity of the object after 8 seconds. Also, use an equation to determine at what time the object will be located 30 metres to the left. The solution of the equation will give two values for the time. If you have difficulty understanding why there are two values determine the velocity that corresponds to each value of the time.

Also see problems 4.7 to 4.10 on page 66.

4.4 Motion on a Surface

Vector quantities are often represented by arrows called vectors.

Fig. 4.5



The order in which vectors are added does not affect the answer. The addition of vectors is commutative.

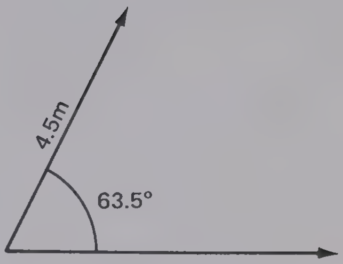
The terms displacement, velocity, and acceleration need not be restricted to describing motion along a line. They can also be used to describe the motion of an object on a surface. If we are describing motion on the surface of the earth it is convenient to describe the direction in terms of compass points.

Consider an object which makes the following changes in its position. It moves 5 metres east, then 4 metres north and finally 3 metres west. This motion may be described by a scale diagram as shown in the sketch in the margin. The vector joining the starting point to the final point represents the *net displacement*. This vector representing the net displacement is the result of three consecutive displacements and is called the *resultant or vector sum* of the other three displacements. Vectors are added then, by setting them down so that the terminal point of one becomes the initial point of the next. The resultant is found by joining the initial point of the first vector to the terminal point of the last vector, as we have seen in our example. The resultant displacement vector in the example is 4.5 metres in the direction 63.5 degrees north of east. Some students are surprised at this result and question how the sum of displacements of 5, 4, and 3 metres could possibly be only 4.5 metres. But remember that this is *vector addition* and the direction as well as the magnitude of the quantities must be considered. This displacement can be measured on the scale diagram or calculated arithmetically.

If this journey took place during a time interval of 10 seconds we could calculate the average velocity.

$\vec{v}_{av} = \frac{d}{\Delta T} = \frac{4.5 \text{ metres}}{10 \text{ s}} = 0.45$ metres per second in the direction 63.5 degrees north of east. Remember this is the *average velocity* which is quite different from the *average speed.*

The addition of vectors may also be illustrated by an example using velocity vectors. Let us calculate the resultant velocity of a boat travelling in a river.

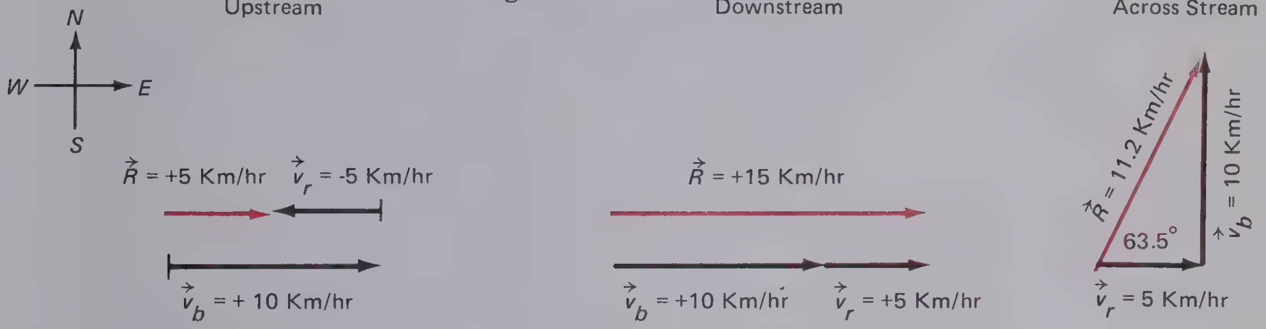


In measuring the angle of the resultant some reference direction must be stated. In this case we have considered the reference direction to be due east and stated the resultant direction as being 63.5° north of *east*. We could have used north as our reference direction and stated our resultant direction as being 26.5° east of *north.*

If a boat, which travels through still water at a speed of 10 kilometres per hour is travelling on a river which flows east with a velocity of 5 kilometres per hour determine its resultant velocity if it heads a) upstream, b) downstream, c) across stream (north).

This problem may be solved by adding the vector representing the velocity of the boat through the water to the vector representing the velocity of the river. These vector additions are shown in Fig. 4.6.

Fig. 4.6


a) Boat Heading Upstream

b) Boat Heading Downstream

c) Boat Heading Across Stream

The resultant velocity may be determined from the scale diagram.

a) Resultant velocity = 5 kilometres per hour west
b) Resultant velocity = 15 kilometres per hour east
c) Resultant velocity = 11.2 kilometres per hour 63.5 degrees north of east.

Note the range in the magnitude of the resultant vectors even though the magnitudes of the vectors added were the same. Both the magnitude and direction are used in vector addition.

The techniques of vector arithmetic are useful in aircraft navigation. The problem arising when flying an aircraft in a wind is similar to those we solved in moving a boat in a river. In the next chapter we shall use the technique of vector addition to determine the net or resultant force when several forces are acting on the same body in different directions.

Q10 A wandering ant sets out on a small journey on a ceiling. He travels 20 centimetres north, 5 centimetres east, 30 centimetres south, 15 centimetres west and then 5 centimetres south. What is his resultant displacement from the starting point?

If the journey took 5 minutes what was the average speed and average velocity of the ant?

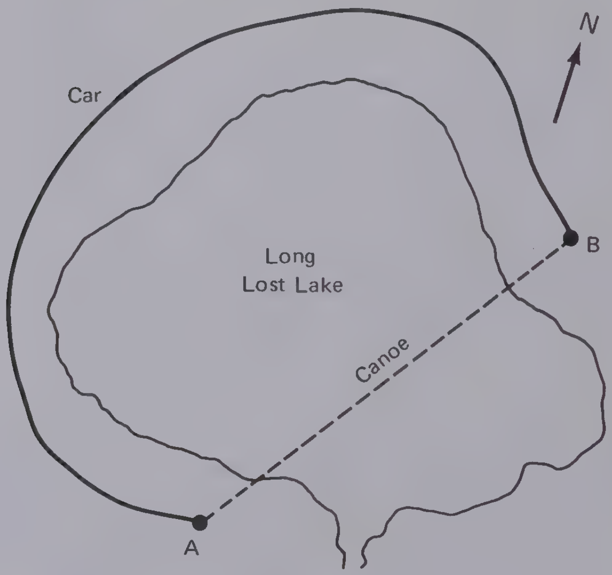
Q11 A bumble bee flies in still air with a speed of 0.5 metres per second. What effect will a 1.0 metre per second wind have on his velocity if he flies

a) into the wind?
b) with the wind?
c) across the wind direction?

Q12 The sketch below represents a journey by a car from A to *B*. According to the odometer the car went 48 kilometres travelling from *A* to *B*. The time taken was 1.2 hours.

Prospector Joe also went from *A* to *B* by canoe in exactly the same time. He found that during his trip, he was always paddling along a straight line exactly 20 degrees east of north. He travelled only 12 kilometres. Both the canoe and the car had the same displacement but the distance travelled by each was different.

a) State the distance travelled and displacement of
 i) the canoe
ii) the car in travelling from *A* to *B*.
(Represent distance by d and displacement by $\vec{d}$.)
b) State the speed and velocity of
 i) the car
ii) the canoe in travelling from *A* to *B*



Also see problems 4.11 to 4.13 on page 67.

Problems Chapter 4

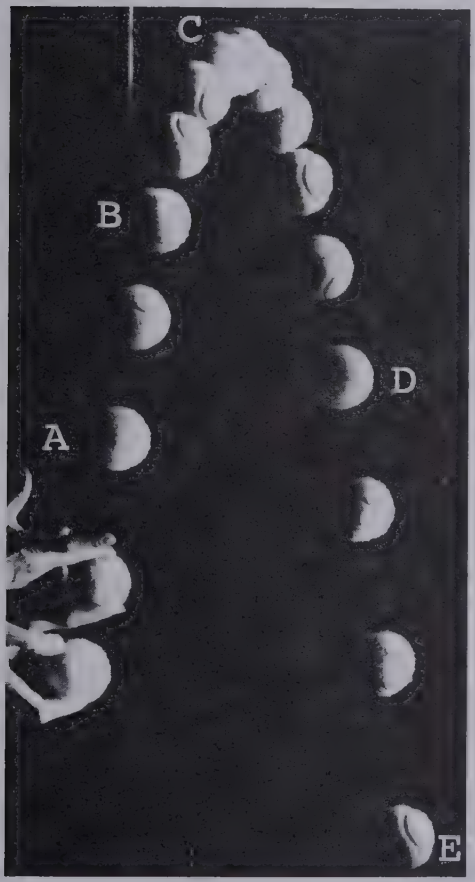
Section A

4.1 Is time a vector or a scalar quantity?

4.2 How could a speedometer be modified to measure velocity?

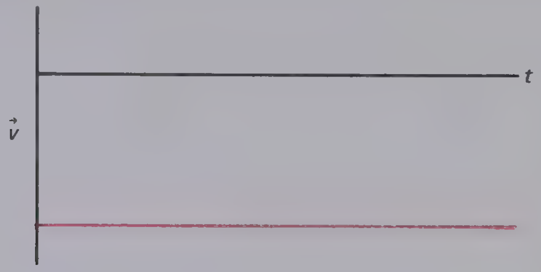
4.3 An object travels west at 10 metres per second for 5 seconds, and then east at 5 metres per second for 4 seconds. What is the displacement of this object from its initial position? What is the object's average speed and velocity?

4.4 The following photograph shows the motion of a ball tossed into the air. The time intervals between successive positions are equal. If the sign convention, upward vectors as positive and downward vectors as negative, is used indicate the sign of the displacement, velocity, and acceleration vectors at the positions indicated. Measure the displacement vector relative to the release point A.

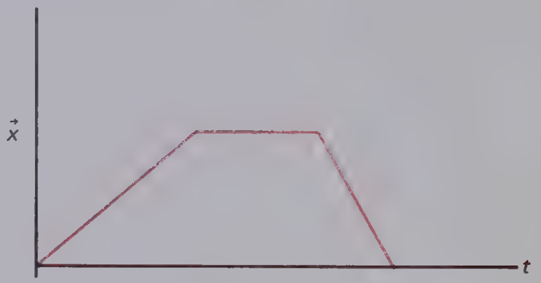


* 4.5 Describe in a few sentences the motions depicted in each of the following graphs. Let all vectors directed to the right be positive.

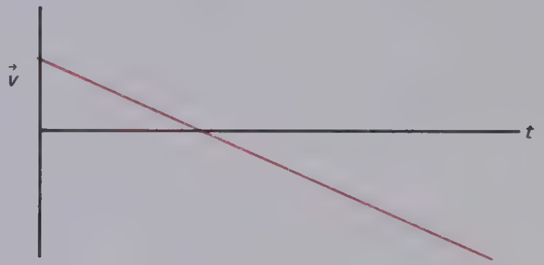
a)



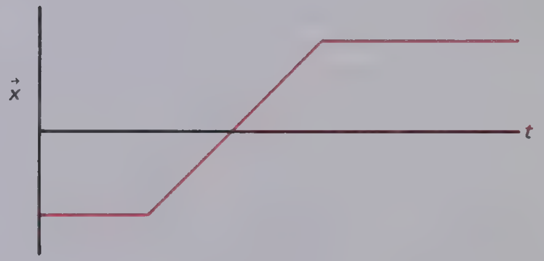
b)



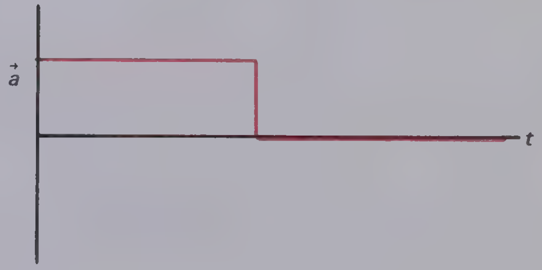
c)



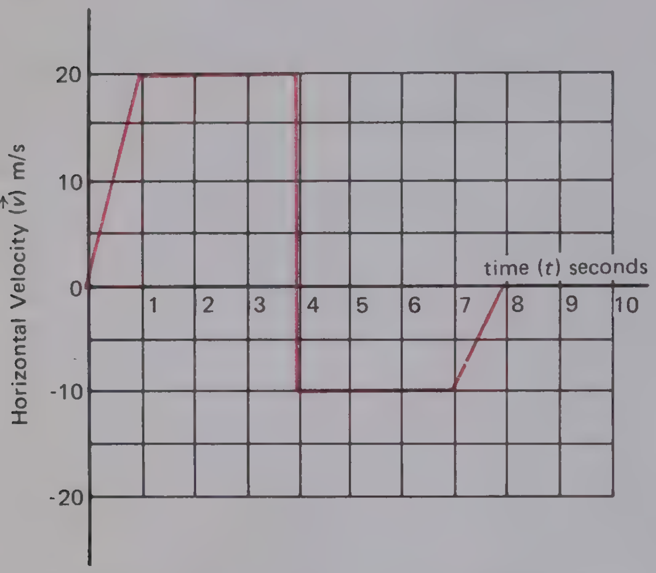
d)



e)



4.6 The graph below shows the horizontal velocity of a ball, which was thrown by a boy, as a function of time.



Discuss the validity of each of the following statements:

a) the ball was in the boy's hand for 1 second.
b) the ball hit an immovable object at the end of 4 seconds.
c) the ball came to rest in a pile of hay.
d) the average horizontal velocity during the flight is 0 metres per second.
e) the ball came back toward the boy for 4 seconds.

4.7 A batter hits a pop fly that travels straight upwards. The ball leaves his bat with an initial speed of 40 metres per second. (Assume $\vec{a}_g$ = 10 metres per second per second.)

a) What is the velocity of the ball at the end of 2 seconds?
b) What is its velocity at the end of 6 seconds?
c) When does the ball reach its highest point?
d) What is the velocity of the ball at the end of 10 seconds?
e) What is its velocity just before it is caught by the catcher?

4.8 A ramp when sloped at a certain angle gives a ball a uniform acceleration of 5 metres per second per second. If a ball is rolled up this ramp at an initial speed of 30 metres per second, how far along the ramp will it roll before stopping?

4.9 A ball starts up an inclined plane with a speed of 4 metres per second, and comes to a halt after 2 seconds.

a) What acceleration does the ball experience?
b) What is the average speed of the ball during this interval?
c) What is the ball's velocity after 1 second?
d) How far up the slope will the ball travel?
e) What will be the velocity of the ball 3 seconds after starting up the slope?

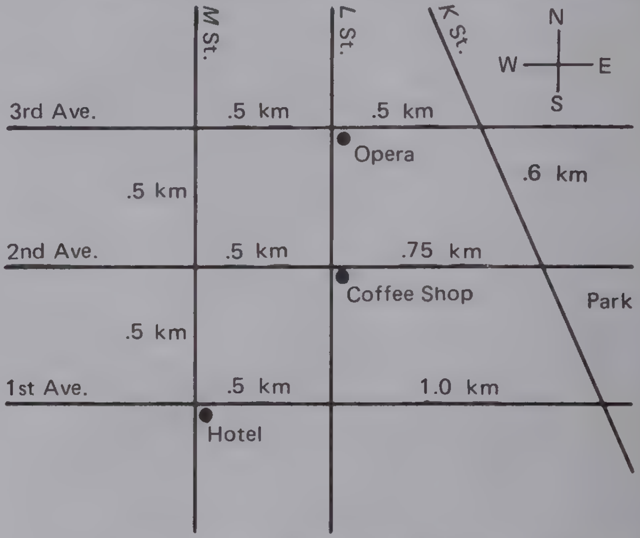
f) What is the total time for a round trip to the top and back to the start?

4.10 A helicopter at a height of 50 metres and rising upwards at 30 metres per second releases an object. ($\vec{a}_g$ = 10 metres per second per second)

a) Sketch the velocity-time graph for the object's motion.
b) What is the highest point reached by the object?
c) At what time does the object reach the ground?

4.11 Now try this. You will need a protractor to measure directions for vector quantities.

Below is a map to scale of downtown Erewhon.



Melvin Tourist gets up and leaves his hotel at the corner of *M* Street and 1st Avenue at 9:00 a.m., heads north along *M* Street, turns right at 2nd Avenue, and ends up at the coffee shop on the corner of *L* and 2nd at exactly 9:15 a.m.

a) What distance did he travel?
b) What was his average speed?
c) What was his displacement?
d) What was his average velocity?

Melvin wandered through Erewhon for the rest of the day and in the evening went to the opera at the corner of *L* and 3rd.

e) How far was he from his hotel now?

After the opera he decided to walk back to the hotel by the scenic route going east along 3rd to *K*, turning right and heading along *K* at a leisurely 3 kilometres per hour.

f) What was his velocity at this time?

At exactly 11:02 p.m. as Melvin crossed 2nd Avenue, an old lady stepped out of the bushes at Happy Park and threatened him with her purse. Melvin ran all the was back to his hotel, going down *K* to 1st and then west to safety. He arrived back at the hotel at 11:17 p.m.

g) What distance did he run?
h) What was his average running speed?
i) What was his displacement from park to hotel?
j) Did he take the shortest route? Explain.
k) What was his total displacement for the day?

$V = d/t$

$d = vt$

4.12 A canoeist, who paddles in still water at 4 metres per second heads across a stream which flows north at 2 metres per second. Determine the time it would take him to travel 100 metres from his starting point if he aims

a) upstream,
b) downstream,
c) across stream.

4.13 If an aircraft pilot aims his plane toward the north and maintains an air speed of 200 kilometres per hour. What would his ground speed and direction of travel be if there was a steady wind at 50 kilometres per hour from the east?

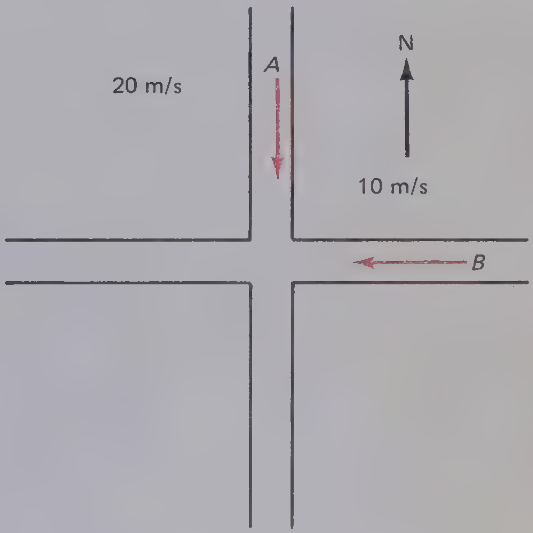
Section B

4.14 The acceleration due to gravity on the surface of the moon is about one-sixth of the value determined on the surface of the earth. If a man can jump to a height of 2 metres on the earth to what height could he jump on the moon? (Assume that the astronaut jumps up from the lunar surface with the same velocity as when he is on the earth. Do you think the assumption is correct?)

4.15 A knight in shiny armour wishes to join a fair maiden located 80 metres across a raging (3 metres per second) river. The knight swims in non-raging water with a speed of 4 metres per second. He knows from a physics course which he took at Knight's school that if he heads straight across the stream he will be carried downstream by the current.

In what direction should he aim if he wishes to travel straight across?

4.16 Car *A* is approaching an intersection 1000 metres away, from the north at 20 metres per second. Car *B* is approaching the same intersection from the east at 10 metres per second and is located 300 metres from the crossing. What is the shortest distance between the two cars? (Hint: Consider the motion of one car relative to the other and use a scale diagram.)



4.17 An ant goes for a long journey in a room. He travels from the ceiling corner in the northeast to the floor corner in the southwest. If the dimensions of the room are 10 m × 10 m × 5 m, determine the displacement of the ant from his initial position.

Chapter 5 *The Birth of Dynamics*
Newton Explains Motion

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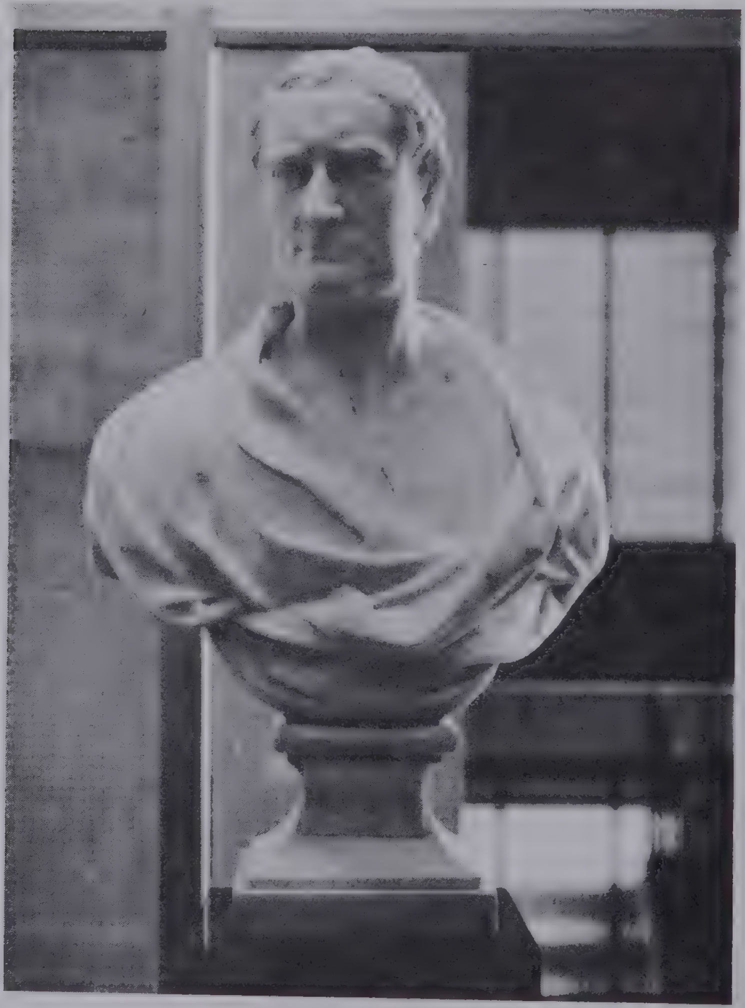


The Birth of Dynamics
Newton Explains Motion

Chapter Five

5.1 "Explanation" and the Laws of Motion

Kinematics is the *study of how* objects move, but not *why* they move. Galileo investigated many topics in kinematics with insight, ingenuity, and gusto. The most valuable part of his work dealt with special types of motion, such as free fall. In a clear and consistent way, he showed how to describe the motion of objects with the aid of mathematical ideas.

When Isaac Newton began his studies of motion in the second half of the seventeenth century, Galileo's earlier insistence that "the present does not seem to be the proper time to investigate the cause of the acceleration of natural motion . . . " was no longer appropriate. Indeed, because Galileo had been so effective in describing motion, Newton could turn his attention to *dynamics*, the study of *why* an object moves the way it does—why it starts to move instead of remaining at rest, why it speeds up or moves on a curved path, and why it comes to a stop.

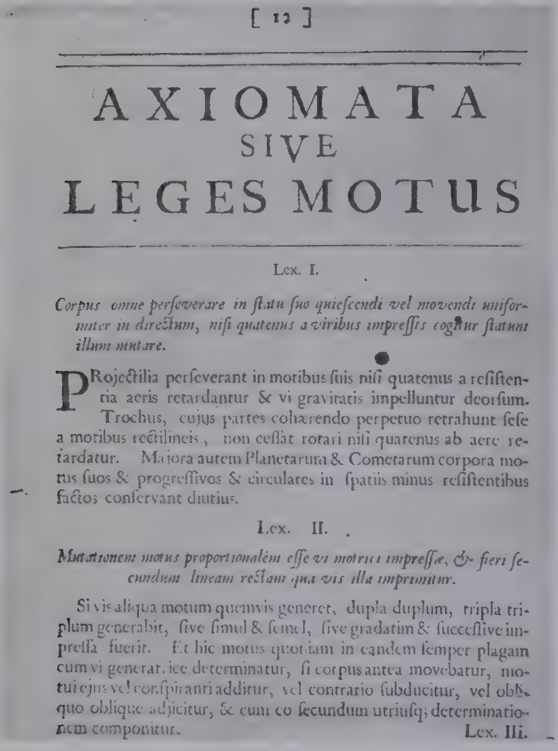
How does dynamics differ from kinematics? As we have seen in the earlier chapters, kinematics deals with the description of motion. For example, in describing the motion of a stone dropped from a cliff, we can write an equation showing how the distance d through which the stone has dropped is related to the time T, during which the stone has been falling. We can find the acceleration and the final speed attained during any chosen time interval. But when we have completed our description of the stone's motion, we are still not satisfied. Why, we might ask, does the stone accelerate rather than fall with a constant speed? Why does it accelerate uniformly as long as air friction is negligible? To answer these questions, we shall have to add to our store of concepts those of *force* and *mass;* and in answering, we are doing *dynamics*. Dynamics goes beyond kinematics by taking into account the cause of the motion.

Kinematic Concepts
- position
- time
- speed
- acceleration

Dynamic Concepts
- mass
- force
- momentum
- kinetic energy

In our study of kinematics we encountered four situations: an object may: (a) remain at rest; (b) move uniformly in a straight line; (c) speed up during straight-line motion; (d) slow down during straight-line motion. Because the last two situations are examples of acceleration, the list could really be reduced to: (a) rest; (b) uniform motion; and (c) acceleration.

Rest, uniform motion, and acceleration are therefore the phenomena we shall try to explain. But the word "explain" must be used with care. To the physicist, an event is "explained" when he can demonstrate that the event is a logical consequence of a law he has reason to believe is true. In other words, a physicist with faith in a general law "explains" an observation by showing that it is consistent with the law. In a sense, the physicist's job is to show that the infinite number of separate, different-looking occurrences all around and within us are merely different manifestations or consequences of some general rules which describe the way the world operates. The reason this approach to "explanation" works is still quite remarkable: the number of general rules or "laws" of physics is astonishingly *small.* In this chapter we shall study three such laws. Taken together with the mathematical schemes of Chapters 2, 3 and 4 for describing motion, they will suffice for our understanding of practically all motions that we can readily observe. And in Unit 2 we shall have to add just one more law (the law of universal gravitation), to explain the motions of stars, planets, comets, and satellites. In fact, throughout physics one sees again and again that nature has a marvelous simplicity.



[12]

AXIOMATA
SIVE
LEGES MOTUS

Lex. I.

Corpus omne perſeverare in ſtatu ſuo quieſcendi vel movendi uniformiter in directum, niſi quatenus a viribus impreſſis cogitur ſtatum illum mutare.

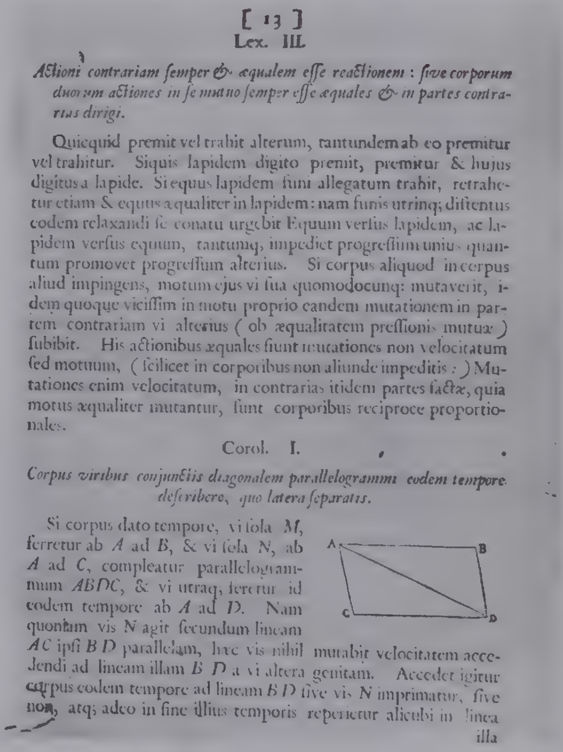
Projectilia perſeverant in motibus ſuis niſi quatenus a reſiſtentia aeris retardantur & vi gravitatis impelluntur deorſum. Trochus, cujus partes cohærendo perpetuo retrahunt ſeſe a motibus rectilineis, non ceſſat rotari niſi quatenus ab aere retardatur. Majora autem Planetarum & Cometarum corpora motus ſuos & progreſſivos & circulares in ſpatiis minus reſiſtentibus factos conſervant diutius.

Lex. II.

Mutationem motus proportionalem eſſe vi motrici impreſſæ, & fieri ſecundum lineam rectam qua vis illa imprimitur.

Si vis aliqua motum quemvis generet, dupla duplum, tripla triplum generabit, ſive ſimul & ſemel, ſive gradatim & ſucceſſive impreſſa fuerit. Et hic motus quoniam in eandem ſemper plagam cum vi generatrice determinatur, ſi corpus antea movebatur, motui ejus vel conſpiranti additur, vel contrario ſubducitur, vel obliquo oblique adjicitur, & cum eo ſecundum utriuſq; determinationem componitur.

Lex. III.



[13]

Lex. III.

Actioni contrariam ſemper & æqualem eſſe reactionem: ſive corporum duorum actiones in ſe mutuo ſemper eſſe æquales & in partes contrarias dirigi.

Quicquid premit vel trahit alterum, tantundem ab eo premitur vel trahitur. Siquis lapidem digito premit, premitur & hujus digitus a lapide. Si equus lapidem funi allegatum trahit, retrahetur etiam & equus æqualiter in lapidem: nam funis utrinq; diſtentus eodem relaxandi ſe conatu urgebit Equum verſus lapidem, ac lapidem verſus equum, tantumq; impediet progreſſum unius quantum promovet progreſſum alterius. Si corpus aliquod in corpus aliud impingens, motum ejus vi ſua quomodocunq; mutaverit, idem quoque viciſſim in motu proprio eandem mutationem in partem contrariam vi alterius (ob æqualitatem preſſionis mutuæ) ſubibit. His actionibus æquales fiunt mutationes non velocitatum ſed motuum, (ſcilicet in corporibus non aliunde impeditis:) Mutationes enim velocitatum, in contrarias itidem partes factæ, quia motus æqualiter mutantur, ſunt corporibus reciproce proportionales.

Corol. I.

Corpus viribus conjunctis diagonalem parallelogrammi eodem tempore deſcribere, quo latera ſeparatis.

Si corpus dato tempore, vi ſola M, ferretur ab A ad B, & vi ſola N, ab A ad C, compleatur parallelogrammum $ABDC$, & vi utraq; feretur id eodem tempore ab A ad D. Nam quoniam vis N agit ſecundum lineam AC ipſi BD parallelam, hæc vis nihil mutabit velocitatem accedendi ad lineam illam BD a vi altera genitam. Accedet igitur corpus eodem tempore ad lineam BD ſive vis N imprimatur, ſive non, atq; adeo in fine illius temporis reperietur alicubi in linea

illa

To explain rest, uniform motion, and acceleration of any object, we must be able to answer such questions as these: Why does a vase placed on a table remain stationary? If an air puck resting on a smooth, level surface is given a brief push, why does it move with uniform speed in a straight line rather than slow down noticeably or curve to the right or left? Answers to these (and almost all other) specific questions about motion are contained either directly or indirectly in the three general "Laws of Motion" formulated by Isaac Newton. These laws appear in his famous book, *Philosophiae Naturalis Principia Mathematica (Mathematical Principles of Natural Philosophy, 1687)*, usually referred to simply as *The Principia*. They are among the most basic laws in physics to this day.

Before we are able to look in more detail at Newton's contributions to motion, we must develop some knowledge of forces and how to work with them.

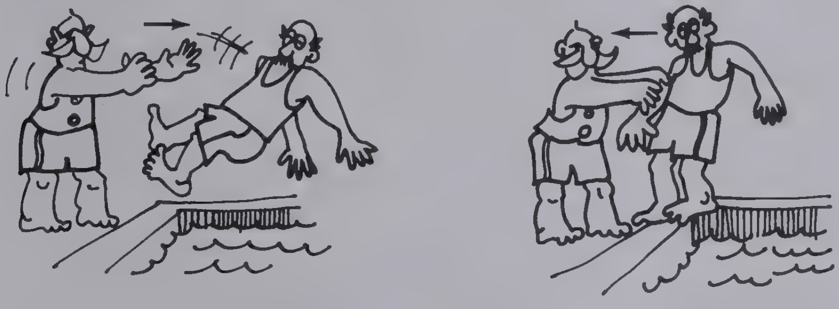
Q1 Distinguish between kinematics and dynamics.
Q2 Would you consider Aristotle as one who concentrated on kinematics, dynamics, or both?

5.2 Forces

Our common-sense idea of force is closely linked with our own muscular activity. We know that an effort or force is required to support a heavy stone. When we push a lawn mower, row a boat, or pull open a door, our muscles let us know we are applying force to an object. **A force is simply a push or pull exerted on an object.**

The effect that a force has depends not only on the magnitude of the force, but also on its direction. In the example shown, both forces have the same magnitude, but the direction in which each is applied differs, and is important to the results.

Both the magnitude and the direction of the force must be stated. Forces are vector quantities.



Force directed right

Force directed left

Fig. 5.1

Forces belong to a class of concepts called *vector quantities*. Vector quantities are those which are defined as having both a magnitude and a direction. Several important physical concepts are vector quantities: force, displacement, velocity, and acceleration. Whenever these concepts are used, both the magnitude and the direction of the quantity should be stated.

Vector Quantities
force
displacement
velocity
acceleration

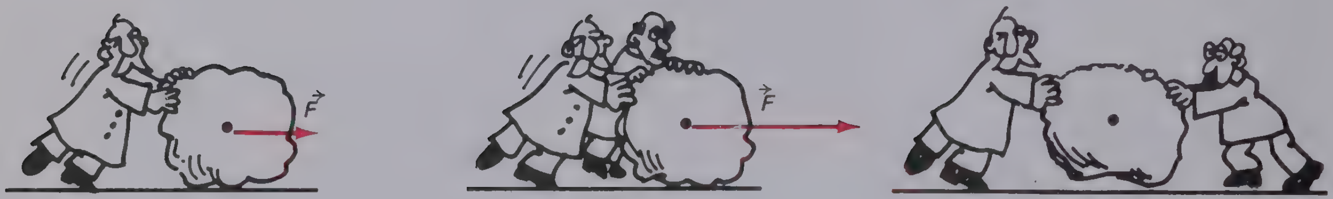
Scalar Quantities
distance
speed
mass
volume
time

Other physical concepts such as, volume, distance, and speed, do not require specification of direction, and so are not vector quantities; these are called scalar quantities. They are represented by *scalars* consisting of a number and an appropriate unit. When you add 10 litres of water to 10 litres of water, the result is always 20 litres, and direction has nothing to do with the result. Similarly, the term speed has no directional meaning; a body moving north might have a speed of 20 metres per second, but its *velocity* should be stated as 20 metres per second [north]. This distinction might appear artificial at first, but it has important consequences.

Q3 How do vector quantities differ from scalar quantities?

5.3 Forces Acting Along a Line—The Sign Convention

We shall confine our discussion of force vectors in this section to forces that act in either direction along a straight line. Consider the situations illustrated in Fig. 5.2



Case 1 Case 2 Fig. 5.2 Case 3

Assume that each boy (well, he is an old boy), exerts the same magnitude of force. Let us call it one unit of force. Then it is reasonable to ask the question: What is the resultant force or net force applied by the boys to the rock in the various circumstances ? To answer this, we must find the sum of the forces applied to the rock by the boys in each case. Since forces are vector quantities, we *must* find the sum of the force vectors, or the *vector sum* of the forces.

We shall introduce a sign convention. Let forces acting to the right be positive and forces acting to the left be negative.

Case 1

$$\vec{F}_{net} = \vec{F}_{boy\ 1}$$

The symbol $\vec{F}$ is used to denote a force vector. The arrow over the $\vec{F}$ reminds us that force is a vector quantity. The $\vec{F}_{net}$ is read net force vector.

Now $\vec{F}_{boy\ 1} = +\ 1\ \text{unit}$

Therefore $\vec{F}_{net} = +\ 1\ \text{unit}$

Case 2

$$\vec{F}_{net} = \vec{F}_{boy\ 1} + \vec{F}_{boy\ 2}$$

$$= (+\ 1\ \text{unit}) + (+\ 1\ \text{unit})$$

This operation sign tells us that the vectors are being *added.*

These signs indicate the *direction* of the force vectors.

$$= +\ 2\ \text{units}$$

Both the magnitude and the direction of a force must be stated.

Thus, the net force exerted by the boys on the rock is 2 units acting to the right. This means that the effect of the two forces exerted by the boys is the same as a single force of 2 units magnitude acting to the right on the rock.

Case 3

$$\vec{F}_{net} = \vec{F}_{boy\ 1} + \vec{F}_{boy\ 2}$$
$$= (+\ 1\ \text{unit}) + (-\ 1\ \text{unit})$$
$$= 0$$

The net force exerted by the boys on the rock is zero!

+ is read "positive" when it indicates direction.

+ is read "plus" when it indicates addition

− is read "negative" when it indicates direction.

− is read "minus" or "less" when it indicates subtraction.

Q4 Determine the net force exerted by the boys in cases 1, 2 and 3 by letting forces acting to the right be negative and those to the left, positive.

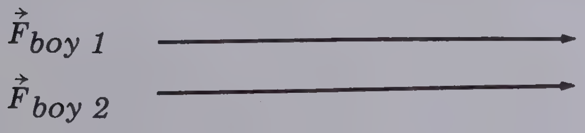
Q5 A body has a force of 5 units acting on it to the right, and a force of 12 units to the left. What is the net force on the body?

5.4 Vector Diagrams

Another convenient way to solve problems involving vector quantities is to represent the quantities by straight arrows called *vectors*, drawn on a page. The arrows are drawn pointing in the direction of the vector quantity, with the length of the arrow representing the magnitude of the quantity.

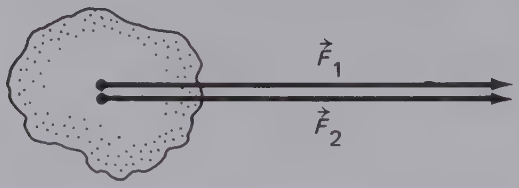
See Experiment 5.4, *Balancing Forces,* pg. 136.

We first pick a convenient scale. Suppose we let the distance 5 centimetres represent a force of "1 unit" in magnitude. The forces exerted by the boys in Case 2 can be represented by vectors 5 centimetres long directed to the right along the page.



Scale:
Let 5.0 cm represent
1 unit of force

It doesn't matter where on the page we draw these force vectors. The same information is conveyed to the reader; namely, that each boy exerts a force of 1 unit magnitude acting to the right. If we want to show that both these forces were applied to the same body, we might do it as indicated in the diagram. ($\vec{F}_{boy\ 1}$ has been shortened to $\vec{F}_1$; similarly $\vec{F}_{boy 2}$ to $\vec{F}_2$.)



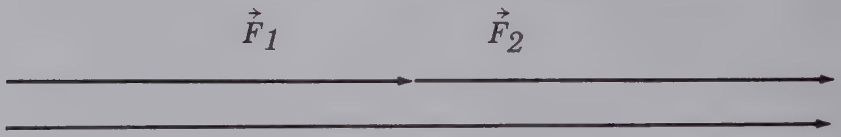
Although both forces actually act on the rock as if they were applied at the same point (the centre-of-mass of the rock), they are drawn slightly separated from each other to emphasize that there are two forces acting.

We reasoned earlier that the resultant or net force exerted by the boys in Case 2 is 2 units to the right. We can add the two force vectors to show a net force of 2 units to the right if we apply the following rule:

The rule for vector addition

Vectors may be added in any order. The tail of a new vector is attached to the head of the preceding one. The resultant vector is always directed from the tail of the first to the head of the last.

Scale: Let 5.0 cm represent
1 unit of force

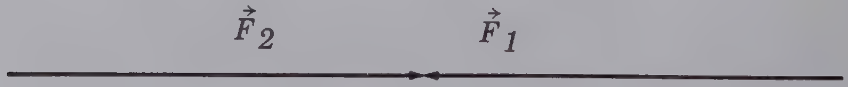


$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$

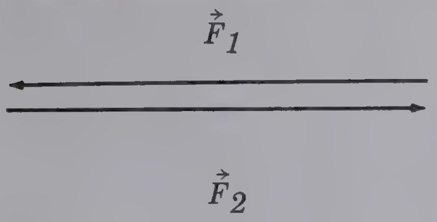
By measurement, the length of the resultant vector is 10.0 centimetres and it is directed to the right.
Therefore,

$$\vec{F}_{net} = 2 \text{ units (right)}$$

We say that this net force vector is the *vector sum* of the two forces $\vec{F}_1$ and $\vec{F}_2$.
Let's try this technique in Case 3:



Adding the vectors using the rule for vector addition



Again the vectors act along the same line. They are drawn slightly separated so that we can see there are two vectors acting.

The line from the tail of the first to the head of the second has no magnitude or direction. Therefore:

$$\vec{F}_{net} = 0$$

Hence, there is no net force acting on the rock.

Whenever the net force acting on a body is zero, we know that the vector sum of all forces acting on the body cancel, or balance each other. This was so in Case 3. In Cases 1 and 2, the net force is not zero and we say that there is an unbalanced force acting on the body. The unbalanced force is just the value of the net force.

The rule for vector addition can even be applied in cases of more than two forces acting in various directions. For example, suppose we want to find the net force when the four forces shown in Fig. 5.3 are applied to the object.

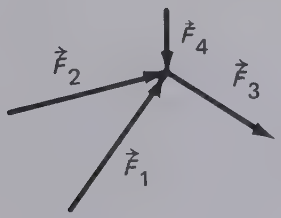
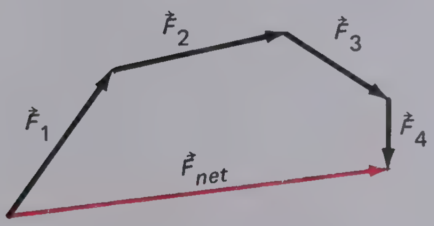
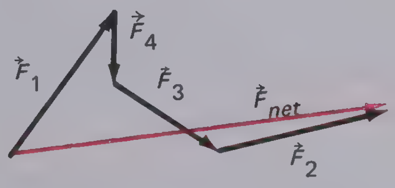


Fig. 5.3

Applying the rule for vector addition:



$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$



$\vec{F}_{net} = \vec{F}_1 + \vec{F}_4 + \vec{F}_3 + \vec{F}_2$

The same net force is achieved regardless of the way in which the vectors are added.

Example Problem

Scale:
Let 10 cm represent
1 unit of force

Three boys are fighting over the possession of a toy. One pulls with a force of 40 units north, another with a 30 unit force east, and the third with a force of 50 units in a direction 53 degrees south of west. What is the net force of the boys on the object? Is it an unbalanced force?

Scale diagram showing the three forces acting on the toy.

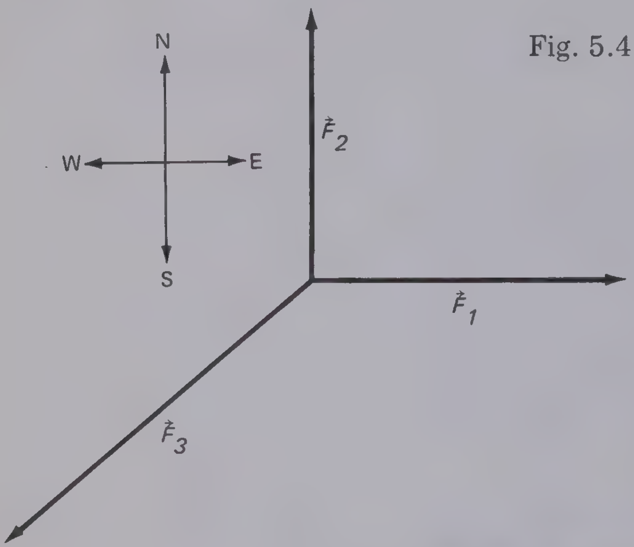
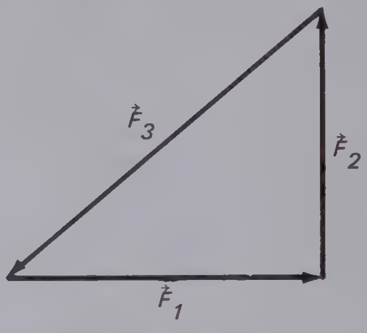


Fig. 5.4

Scale diagram showing vector addition.

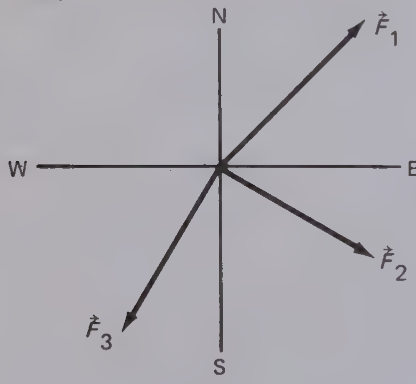


$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

This vector diagram is drawn to the same scale as Fig. 5.4. The three force vectors are being added together in this diagram.

The resultant of the vector sum of the three forces acting on the toy, the net force vector, is found by joining the head of the last vector drawn ($\vec{F}_3$) to the tail of the first ($\vec{F}_1$). But, in this example, this line has no magnitude or direction. The net force is zero. There is no "unbalanced" force on the toy.

Scale: Let 1 cm represent 3 units of force.



Q6 State the rule for vector addition.

Q7 Explain in your own words what you are finding when you determine the net force acting on an object.

* **Q8** Use a protractor to state the magnitudes and directions of the force vectors drawn in the margin.

Also see problems 5.1 to 5.3 on page 92.

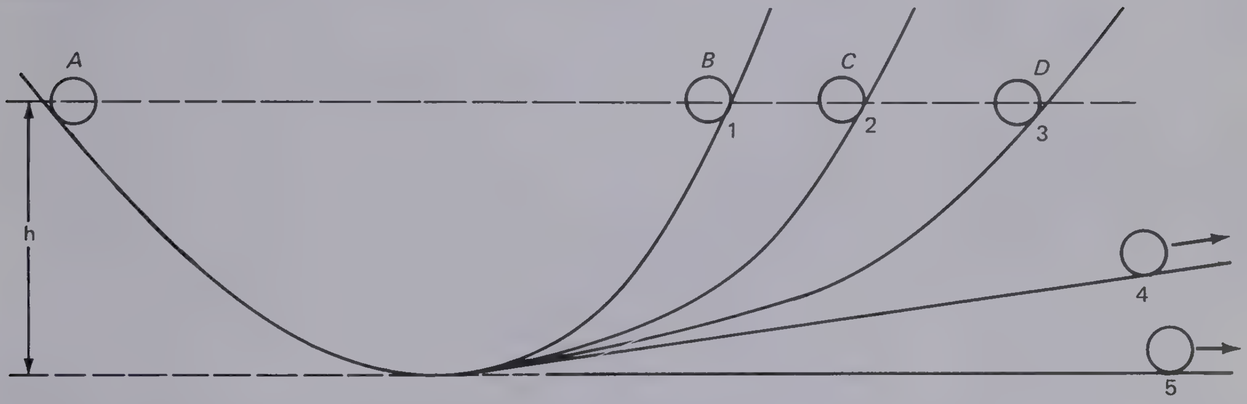
5.5 Newton's First Law of Motion

Before you read this section you might like to try Experiment 5.1, pg. 131.

Now that you have the skills to enable you to determine the net force acting upon objects, you will be able to understand our modern explanation for the causes of motion, and see what was needed in order to overthrow the Aristotelian theories which had been accepted for over 2000 years.

Although Newton was first to formally enunciate the statement that is known as Newton's first law of motion, Galileo provided the line of reasoning that led to the downfall of the older system.

Galileo proposed the following thought-experiment. A ball is released from a height h above the bottom of a perfectly frictionless ramp. Suppose that it can then roll up other ramps that are also frictionless. Galileo predicted that the ball will rise to the same height h on the other ramp, no matter what the slope of this other ramp is! For example, according to Galileo, a ball released at point A, will rise to points B, C, and D as the incline is lowered.



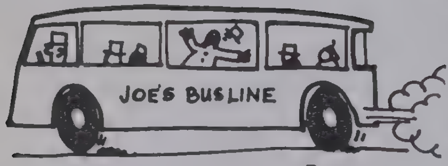
We notice that the horizontal distance the ball must roll in order to reach the height h becomes greater as the incline changes from positions 1 through 3. In an important insight Galileo pointed out that if the incline is *very* slight, the ball will move *very* far horizontally until it reaches height h. Finally, if there is no incline at all (see position 5 in the diagram), the ball should keep moving along the line at constant velocity forever! Remember, a body moving at constant velocity will change neither its speed nor its direction. The ball, while moving at constant velocity along the flat surface, receives no pushes or pulls at all in the direction parallel to the surface since there is no friction or air resistance in this thought-experiment. The net force on the ball is zero, and yet it moves!

Newton, fifty years later, developed Galileo's reasoning further, and formally stated what is accepted as a correct explanation of motion at uniform *velocity.* Newton's first law of motion is stated as follows:

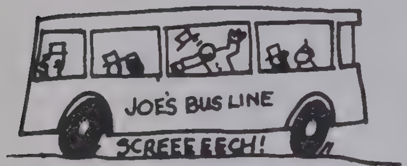
> **Every object continues in a state of rest or of uniform motion in a straight line unless acted upon by an external unbalanced force.**

This statement tells us that every body resists any changes in its speed or direction of motion. If at rest, the body will remain at rest. If moving at constant velocity, it will continue to do so in the absence of any external unbalanced forces. This property of maintaining velocity unchanged is an inherent property of all objects, and is called **inertia.** The first law is often termed the law of inertia because it is simply a statement that describes the property called inertia.

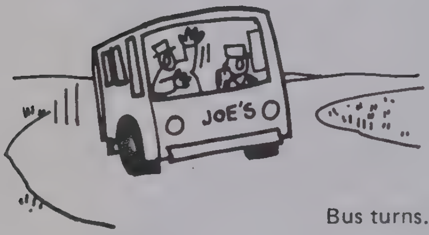
Many common occurrences illustrate the inertia of bodies. In a bus, you appear to be thrown backward as the bus begins to move forward. Your inertia tends to keep you at rest, with respect to the ground, while the bus drives out from under you. If the bus turns a corner to the right, you seem to be thrown to the left of the bus. But you are not actually being pushed, or thrown to the left. This is just your tendency to keep moving in a straight line. You will continue to do so until something, perhaps the window on the left side of the aisle, exerts an unbalanced force on you to make you change direction. As the bus slows to a stop, your tendency is to keep moving forward. You feel as if you are being thrown to the front of the bus. When you shovel snow, you first get the shovel and the snow moving, then stop the shovel with your arms. The snow continues in motion, exhibiting its property of inertia. A curling stone gliding down the ice and a motor boat drifting forward after its engine is shut off are displaying their tendency to maintain their velocity unchanged. Note that inertia is not a force, nor is it capable of exerting a force, but it merely describes a property of all objects.



Bus starts.



Bus stops.



Bus turns.

Q9 Compare the Newtonian and Aristotelian explanations of motion at constant velocity.

Q10 What is inertia?

Q11 List some examples of inertia (other than those mentioned in the text) exhibited by

i) stationary bodies

ii) moving bodies

Also see problems 5.4 to 5.6 on page 92.

5.6 Further Notes on the Law of Inertia

See *Inertia Demonstration,* Activities 5.1 to 5.5, pgs. 138 and 139.

You may have found Galileo's thought experiment to be convincing, but remember that neither Galileo nor Newton proved the principle of inertia. Think of how you might try to verify that principle experimentally. You could start an object moving (perhaps a frictionless air puck) in a situation in which there is no unbalanced force acting on it. Then you could observe whether or not the object continued to move uniformly in a straight line, as the first law claims it should. But there are at least two drawbacks to this experiment:

1. How do you know that there is no unbalanced force acting on the object? The only answer we have is: because the object continues to move uniformly in a straight line. But that reason is merely a restatement of the first law which we wanted to prove by experiment. Surely we cannot use the first law to verify the first law!

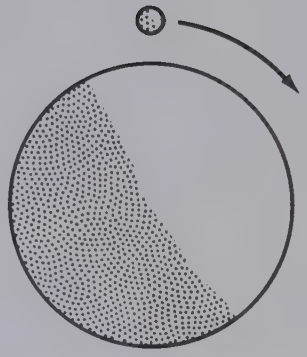
Fasten your seatbelts, fight Inertia!

2. What is meant by a "straight line"? When Galileo wrote about uniform motion along a "straight" line, he was thinking of a line parallel to the earth's surface. Thus, an object given an initial push would move around and around the earth. Galileo's

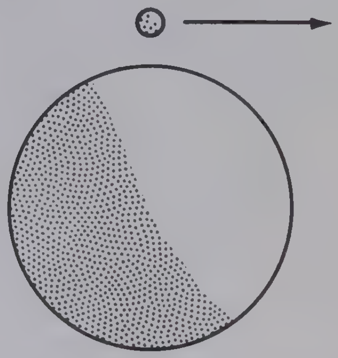
"straight line" was really a circle! Newton, on the other hand, meant an ideal straight line such as you define in plane geometry. Such a line is not parallel to the earth's surface; instead it continues indefinitely out into space. The line traced by an air puck in any actual laboratory experiment is so short relative to the earth's circumference, that it could fit either Galileo's or Newton's meaning.

What then is the significance of Newton's first law of motion? For convenience let us list the important insights the first law provides.

1. It presents inertia, the tendency of an object to maintain its state of rest or uniform motion, as a basic inherent property of all objects.
2. It makes no dynamical distinction between an object at rest and an object in uniform motion. Both states are characterized by the absence of unbalanced forces.
3. It is a general law. It emphasizes right from the start that a single scheme is being formulated to deal with motion anywhere in the universe. For the first time no distinction is made between terrestrial and celestial domains. The same law applies to objects on earth as for planets and stars.
4. It raises the whole issue of reference frames. An object stationary for one observer might be in motion for another observer; therefore, if the ideas of rest or uniform motion are to have any significance, a reference frame must be stipulated.
5. The first law informs us of the behaviour of objects when no force acts on them. Thus, it sets the stage for the question: what happens when a force does act on an object?



Galileo's idea of a straight line.



Newton's idea of a straight line.

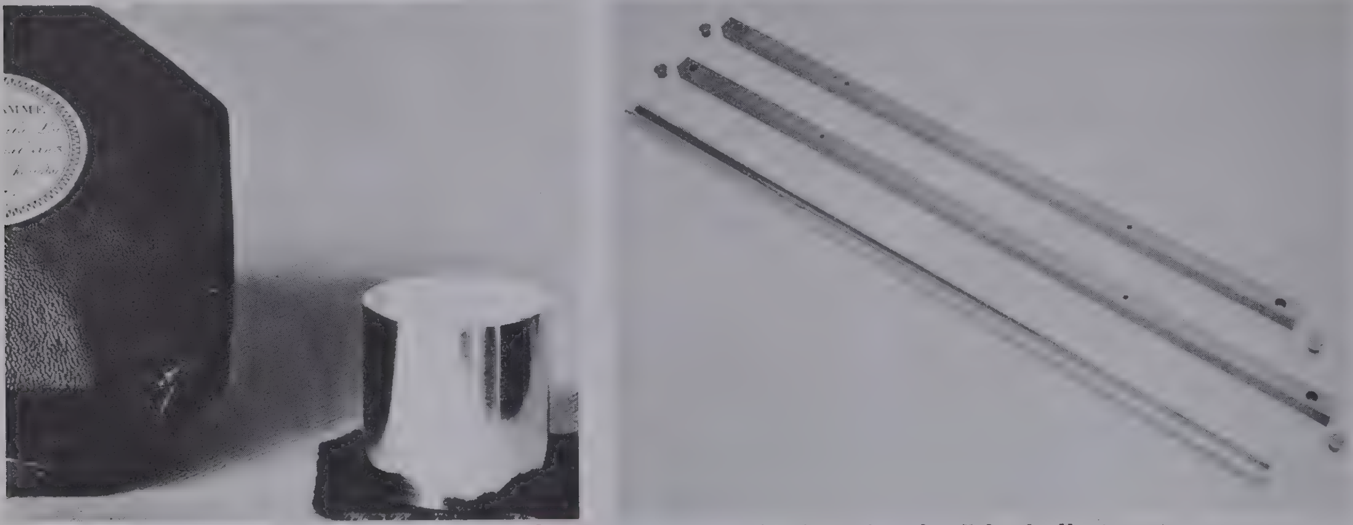
5.7 Mass: A Fundamental Concept

In section 5.5, we introduced inertia as the name given to describe a manner in which all bodies behave. When we actually measure how much of this property one body has with respect to others, we speak of the mass of the body. Therefore, mass is a measure of the inertia of a body.

Impt.

The idea that bodies have a mass has been part of physics for a long time, and originally the word was thought of as somehow representing the amount of material that was in a body. This statement is not precise enough to be accepted now as a definition of mass, but it will help us see how a unit with which to measure mass can be selected. One way to do this is to choose some convenient object, perhaps a piece of corrosion-free metal, as the universal standard of mass, just as a metre is a universal standard of length. We can arbitrarily assign to this object a mass of one unit. Once this unit has been selected we can proceed to develop a measure of force.

Although we are free to choose any object as a standard of mass, ideally it should be exceedingly stable, easily reproducible, and of reasonably convenient magnitude. Such a standard object has, in fact been agreed on by the scientific community. By international agreement, the primary standard of mass is a cylinder of platinum-iridium alloy, kept near Paris at the International Bureau of Weights and Measures. The mass of this platinum cylinder is defined as exactly 1 kilogram (abbreviated 1 kg). Accurately made copies of this international primary standard of mass are kept in the various standards laboratories throughout the world. Further copies have been made from these for distribution to manufacturers and research laboratories.



The standard kilogram and metre are kept near Paris at the International Bureau of Weights and Measures. No standard pound exists. A pound mass is defined as 0.4535924 kg

Q12 How does the inertia of a 5 kg ball at rest compare with the inertia of a 5 kg ball in motion?

5.8 Newton's Second Law

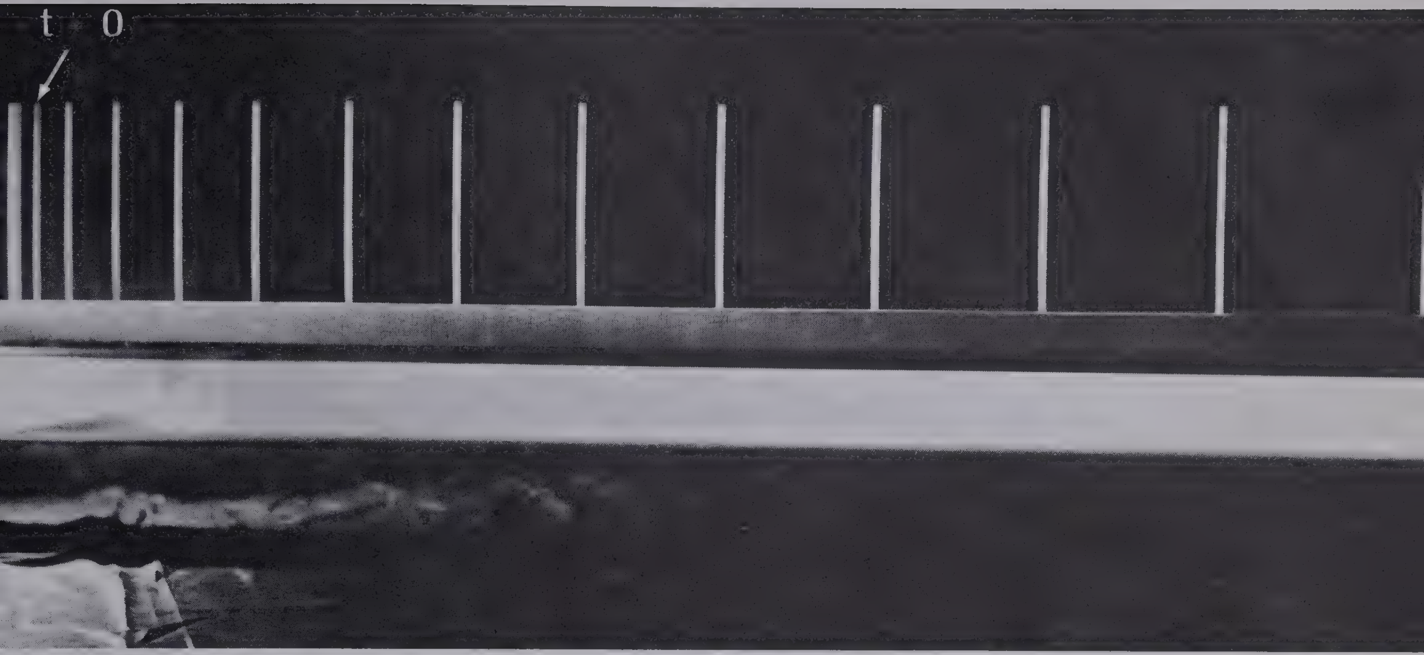
Newton's first law tells us that the velocity of a body does not change when the forces acting on the body are balanced. Because its velocity does not change, we can say that its acceleration is zero.

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = 0$$

What type of motion results when a constant unbalanced force acts on a body? An analysis of the motion in the photo provides the answer.

To produce motion, a constant force was exerted on a glider free to move along a nearly frictionless path. The force was the gravitational force provided by the small mass suspended over the edge of the table and connected to the glider by a string hung over a pulley. A straw attached to the glider served to mark the position of the glider after each interval flash of the stroboscopic light.

Since the distances travelled in each time interval are not equal, we know that the velocity of the puck changes: the puck is accelerating. To decide if the acceleration is constant or if it changes in some way, let us use a technique developed in Section 2.7 of Chapter 2.



The first step is to measure the distances travelled from time $t = 0$ to the end of each succeeding time interval.

This data is recorded in Table 5.1.

Table 5.1

Time Elapsed from time $t = 0$ (flashes)	Distance Covered since time $t = 0$ (mm)
0	0
1	4.0
2	9.5
3	17.0
4	26.2
5	37.6
6	51.0
7	66.0
8	82.5
9	101.2
10	122.0
11	143.8
12	168.8

This data has been plotted in Fig. 5.5. We can use this distance-time graph to determine the instantaneous speeds of the body at any chosen time by drawing a tangent to the curve at that time and measuring the slope of the tangent. This has been done on the graph. The value of the instantaneous speed at several different times has been recorded in Table 5.2.

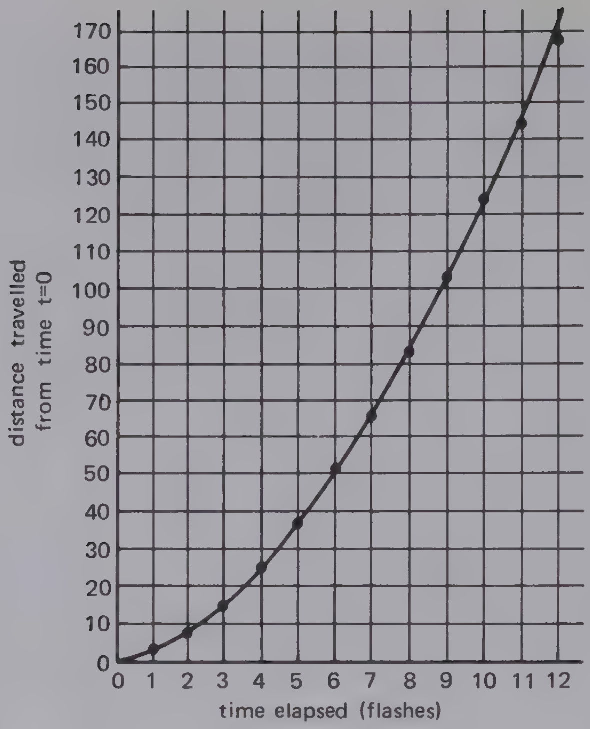
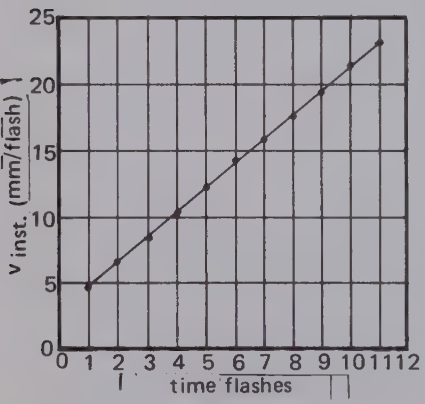


Fig. 5.5

Table 5.2

Time Elapsed from time $t = 0$ (flashes)	Instantaneous Speed V_{inst} (mm/flash)
1	4.8
2	6.6
3	8.5
4	10.3
5	12.2
6	14.1
7	15.9
8	17.7
9	19.5
10	21.4
11	23.2
12	cannot determine slope

Fig. 5.6



Finally, these values of instantaneous speed have been plotted against the time at which they occur (see Fig. 5.6). This graph is a straight line, indicating a constant acceleration for the glider. **We may conclude that a constant unbalanced force acting on a body produces a constant acceleration in the direction of the force.**

The application of larger unbalanced forces to the same mass results in new constant accelerations that bear a simple relationship to the values of the unbalanced force applied. Table 5.3 shows the results of an experiment in which accelerations produced by a number of different forces were measured.

Table 5.3

Unbalanced Force on Object	Mass of Object (kg)	Acceleration Produced (m/s²)
1	2	3
2	2	6
3	2	9
8	2	24

Units for force will be discussed in the next section.

Newton summarized this behaviour by saying that the acceleration produced on a given body varies directly with the unbalanced force. In symbols:

$$\vec{a} \propto \vec{F}_{unbalanced}$$

To write this statement as an algebraic equation rather than as a proportionality statement requires the choice of certain specific units for expressing force, mass, and acceleration. This choice is discussed in the following section.

But the acceleration produced by any one value of the unbalanced force depends also on the mass of the body being accelerated. Table 5.4 summarizes how the accelerations depend on mass.

Table 5.4

Unbalanced Force on Object	Mass of Object (kg)	Acceleration Produced (m/s²)
1	1	6
1	2	3
1	3	2
1	5	1.2

This behaviour is described by saying that the acceleration produced varies inversely with the mass of the object. In symbols:

$$\vec{a} \propto \frac{1}{m}$$

We can combine the roles played by force and mass in the second law into a single statement:

> **An object that is acted on by an unbalanced force experiences an acceleration in the direction of the force. The acceleration varies directly as the force and inversely as the mass of the object.**

Fortunately, the idea expressed in this long statement can be summarized in the equation:

$$\vec{a} = \frac{\vec{F}_{unbalanced}}{m}$$

This relationship is often expressed in the form

$$\vec{F}_{unbalanced} = m\vec{a} \text{ or } \vec{F}_{net} = m\vec{a}$$

In either form, this is probably the most fundamental equation in all of Newtonian mechanics. Like the first law, the second has an incredible range of application. It makes no difference whether the force is mechanical, electric, or magnetic, whether the mass is that of a star or of a nuclear particle, whether the acceleration is large or small. We can use the law in the easiest problems and in the most sophisticated ones. By measuring the acceleration which a known force gives a body of *unknown mass*, we can compute a numerical value for the mass from the equation $m = \vec{F}/\vec{a}$. Clearly, we must be able to measure two of the three quantities, mass, net force, and acceleration, in order to compute the third.

Q13 State Newton's second law (a) in words (b) in equation form.
Q14 "Newton's second law holds only when frictional forces are ignored". True or false? Discuss.
Q15 The same unbalanced force is applied to bodies of masses 1 kg, 3 kg, 5 kg, 10 kg. If the acceleration produced on the first body is $15\frac{m}{s^2}$, what will be the accelerations of the other bodies?
Q16 To push a book at constant velocity along a table requires a constant applied force. What is the value of the net force on the book?

5.9 Units of Force

We have introduced and discussed the concept of force in this chapter, but have said nothing specific about the units in which we might express values.

You can verify Newton's second law by trying Experiment 5.2, pg. 132.

The simplest way to define a force unit would be to take a standard kilogram mass and pull it along a level, frictionless surface, adjusting the amount of the force until the 1 kilogram mass accelerates at 1 metre per second per second. Then call the force being exerted 1 unit of force, and give the unit a name. We can express this mathematically as follows.
According to the second law

$$\begin{aligned} \vec{F}_{net} &= m\vec{a} \\ \text{1 unit of force} &= 1\ \text{kg} \times 1\ \text{m/s}^2 \\ &= 1\ \frac{\text{kg-m}}{\text{s}^2} \end{aligned}$$

Thus, 1 kilogram-metre per second per second is that net force which will give a mass of 1 kilogram an acceleration of one metre per second per second.

This unit, a kilogram metre per second per second, is awkward to say, and it has been given the shorter name, one *newton*. The newton then, is a *derived unit* which has been defined in terms of the fundamental units in the MKS system, the *m*etre, the *k*ilogram, and the *s*econd.

Q17 A net force of 10 N gives an object a constant acceleration of 4 m/s^2. What is the mass of the object?

Q18 A mass of 15 kg on a frictionless surface is given an acceleration of $3\frac{m}{s^2}$ by an applied force. What was the value of the force?

* **Q19** Why is the newton a *derived* unit, whereas the kilogram and the metre are *fundamental* units?

See also problems 5.7 to 5.13 on page 92.

Newton's third law is developed by considering an explosion in Experiment 5.3, pg. 134.

5.10 Newton's Third Law

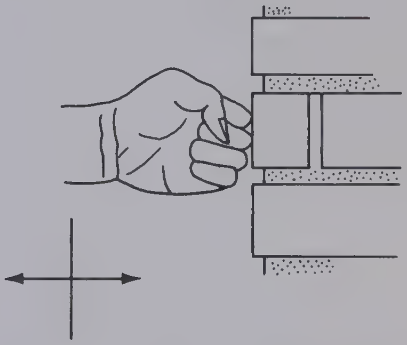
The third major contribution Newton made to our understanding of dynamics does not concern motion directly, but points out a different consequence of forces.

Strike your hand against a wall—gently at first, then harder! Your hand feels forces on it—and these forces apparently come from the wall. You have experienced this feeling before, of course, and perhaps you are not surprised by what happened, but you should be! After all, you were told to hit the wall, not to have the wall hit your hand. Apparently, when you exert a force on some other object, that object exerts a force back on you.

This kind of interaction can even be detected when there is no physical contact between the two objects being considered! Admittedly, the kinds of forces involved must be those that can act between bodies even when they are not touching. You are probably familiar with at least one of these, magnetic forces.

If you hold a small magnet in your hand and let it approach a sample of iron or nickel, you can see the effect of the magnet on the sample because it will move, and you can *feel* the effect of a force exerted by the sample on the magnet in your hand.

Newton used the results of experiments done by others to show that a simple relationship between these forces exists in every case. We come now to an explicit statement of Newton's third law.



The force that the wall exerts on the hand is equal in magnitude but opposite in direction to the force that the hand exerts on the wall.

> **If object A exerts a force on object B, then object B exerts a force equal in magnitude but opposite in direction on object A.**

In algebraic form

$$\vec{F}_{A\ on\ B} = -\vec{F}_{B\ on\ A}$$

Newton's third law shows clearly that:

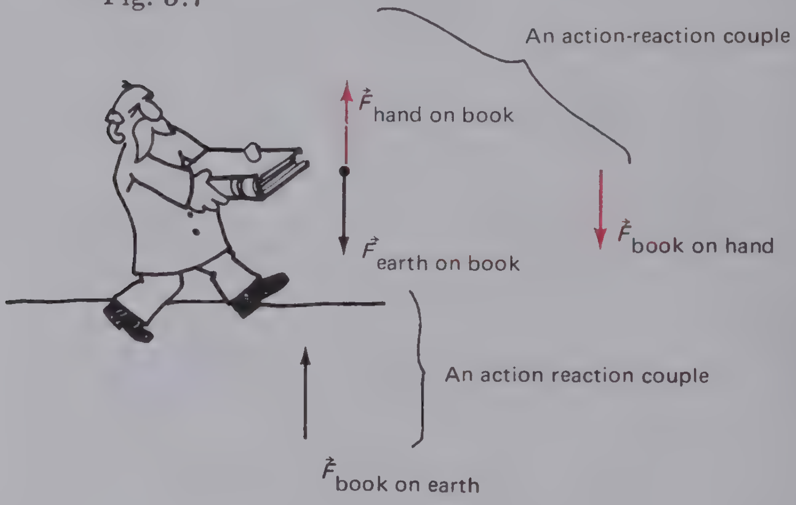
1. forces always occur in pairs;
2. each force of the pair acts on a *different* object;
3. each force of the pair is equal in magnitude but acts in the opposite direction to each other.

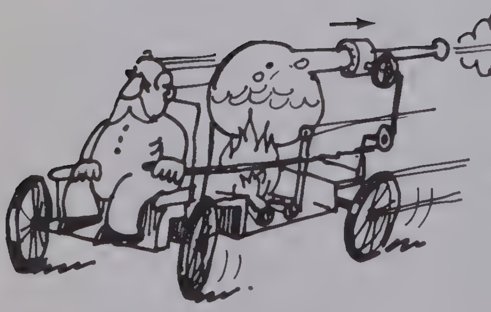
We often refer to one force in the pair as an *action* force and the other as a *reaction* force. The pair of forces is called an *action-reaction* couple. If these three points are always kept clearly in mind, then a shorter statement of the third law is acceptable.

> **For every action (force) there is an equal and opposite reaction (force).**

Newton's third law differs from the first two laws in that it is not a law which describes a connection between force and motion. It is solely a statement about forces, and makes no mention of the state of motion of bodies. It tells us, for instance that if the earth exerts a "gravitational pull" down on a man, then the man exerts an equal gravitational pull up on the earth. This is true regardless of any other forces that may be acting on either the earth or the man. Notice also, that the state of motion of one body in an action-reaction pair is not affected in any way by the existence of the reaction force which acts on the other body. The net force on the book in Fig. 5.7 is zero because the downward force of gravity acting on the book is just balanced by the upward force of the hand on the book. Both these forces act on the same object, the book, and are *not* an action-reaction couple.

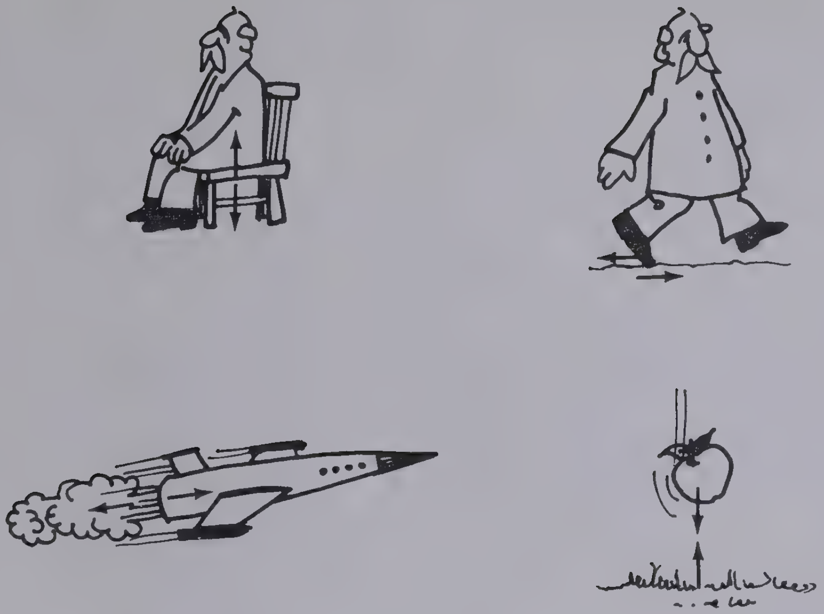
Fig. 5.7


An action-reaction couple
$\vec{F}_{\text{hand on book}}$
$\vec{F}_{\text{book on hand}}$
$\vec{F}_{\text{earth on book}}$
An action reaction couple
$\vec{F}_{\text{book on earth}}$



Q20 State Newton's third law.
Q21 What is an action-reaction couple?
Q22 Why is Newton's third law not involved with the motion of a body?
* Q23 This vehicle was designed by Isaac Newton. Account for its forward motion.

See problems 5.14 to 5.17 on page 93.



Action-Reaction Couples

5.11 Weight and Mass—A Comparison

Many people mistakenly think that mass and weight have the same meaning. In fact, in everyday applications, an object's mass and its weight have the same values. However, the two terms represent different concepts. They are neither defined nor measured in the same way. Let us examine these differences.

Wrong!

The weight of a body is equal to the magnitude of gravitational force acting on it.

The mass of a body is a measure of its inertia.

We can determine the gravitational force on a body by combining Newton's second law with our knowledge that all bodies fall to earth with the same acceleration (ignoring air friction), of 9.8 metres per second per second. By Newton's second law,

$$\text{Force}_{\text{net}} = \text{mass} \times \text{acceleration}$$

But the force which produces the acceleration of a freely falling body is the force of gravity, F_g.

Therefore,

$$\vec{F}_g = m\vec{a}_g$$

But, the force of gravity on a body of mass m is by definition, the weight of the body.
Therefore,

$$\text{Weight of mass m} = ma_g$$

The weight of a 1 kilogram mass on earth for example, is

We omit the direction of this force because weight is the magnitude of the force of gravity only.

$$F_g = 1\text{ kg} \times 9.8\ \frac{\text{m}}{\text{s}^2}$$
$$= 9.8\text{ newtons}$$

The value of the free fall acceleration of a body, the acceleration due to gravity, varies slightly from 9.8 metres per second per second at certain places on the earth's surface. This means that the weight of the body would be slightly different at these places. A 1 kilogram object would weigh the amounts shown in Table 5.4 at each of the places listed. These differences are due to irregularities in the composition of the earth's crust and to the fact that the earth is not completely spherical, since its diameter at the equator is greater than its diameter from north to south poles.

Table 5.4 Variation in the value of $\vec{a}_g$ on earth.

Place	North latitude	Elevation (m)	Free Fall Acceleration $\left(\frac{\text{m}}{\text{s}^2}\right)$
Canal Zone	9°	0	9.78243
Jamaica	18°	0	9.78591
Bermuda	32°	0	9.79806
Denver	40°	1638	9.79609
Cambridge, Mass.	42°	0	9.80398
Greenland	70°	0	9.82534

The magnitude of the free fall acceleration of all bodies on the surface of the moon is 1.6 metres per second per second, only 1/6 of that on the earth. The same 1 kilogram mass on the moon would weigh

$$F_g = ma_g\ (\text{moon})$$
$$= 1\text{ kg} \times 1.6\ \frac{\text{m}}{\text{s}^2}$$
$$= 1.6\text{ newtons}$$

The values of the weight of a 1 kilogram mass on other planets in our solar system are listed in Table 5.5.

Table 5.5 Variation in the weight of a 1 kg object if place on different planets.

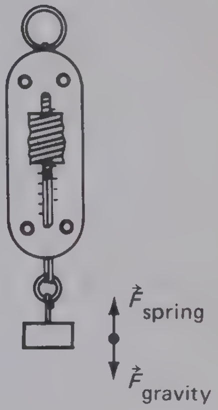
Planet	Weight of a 1 kg mass
Jupiter	26.2 N
Neptune	14.0 N
Saturn	11.2 N
Uranus	10.6 N
Earth	9.8 N
Venus	9.0 N
Mercury	3.9 N
Mars	3.8 N
Pluto	uncertain
Moon	1.6 N

The mass of body never changes, however, and its value does not depend on the value of the force of gravity in any way.

You can get a fairly good idea of the difference between the properties of weight and mass by holding a big book: First, just lay the book on your hand; you feel the weight of the book acting down. Next, grasp the book and shake it back and forth sideways. You still feel the weight downwards, but you also feel how hard the book is to accelerate back and forth. That is, you experience its inertia, or mass. You could make your sensation of the book's weight disappear by hanging the book on a string, but the sensation of its inertia as you shake it remains the same. This is only a crude demonstration, and it is not clear that the shaking sensation does not still depend on the pull of the earth. More elaborate experiments would show, however, that weight can change without changing mass. Thus when an astronaut on the moon's surface uses a big camera, he finds it much easier to hold—its *weight* is only 1/6 of its weight on earth. But its *mass* or inertia is not less, and it is as hard to swing around suddenly into a new position as it is on earth.

Measuring Weight

One direct way of measuring weight is to suspend the body from a coiled spring. The object will be pulled down by gravity until the force of the spring pulling up on it is equal to the gravitational force pulling down. The value of the force exerted by the spring, which is read from the pointer on the scale, is thus a measure of the weight of the body.



Spring balance used for measuring weight.

Measuring Mass

Using an *equal arm balance* is one way to measure the mass of an unknown object in terms of the standard mass, the kilogram.

If the balance is level, then the force of gravity pulling down on the standard masses on the right must be equal to the force of gravity on the unknown object on the left.

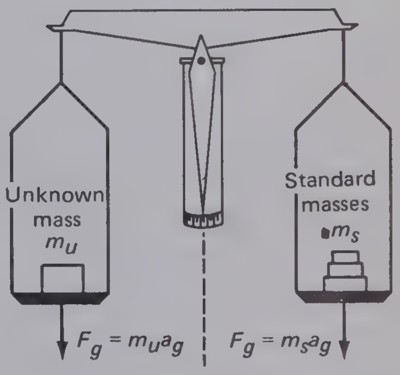
Therefore, $F_{\text{unknown}} = F_{\text{standard mass}}$

and $m_u a_g = m_s a_g$ (equal weights)

but since all bodies have the same free fall acceleration, $\vec{a}_g$, we arrive at

$$m_u = m_s \quad \text{(equal masses)}$$

and we can see that the unknown mass, m_u, is equal to m_s, the known mass.



The equal arm balance is used to measure mass.

Q24 Suppose the equal arm balance with the same unknown mass on it were used on the moon. Would the measurement of the mass of the object be the same as it is on the earth? Why?

Q25 An astronaut is orbiting the earth in a space vehicle. The acceleration due to gravity at that distance is one half of that on the surface of the earth. Which of the following are true?

(a) His weight is zero.
(b) His mass is zero.
(c) His weight is half its original value.
(d) His mass is half its original value.
(e) His weight remains the same.
(f) His mass remains the same.

Q26 A boy jumps from a table top. When he is halfway between the table top and the floor which of the statements in Q25 are true?

Q27 List as many differences as you can between the mass and the weight of a body.

See also problems 5.18 to 5.22 on page 93.

5.12 An Apparent Loss in Weight

On some elevators, especially those that make non-stop trips from the ground floor to the top floor of tall buildings, passengers are aware of strong sensations during the times of acceleration. As the elevator begins to descend, the passengers feel lighter than usual, their arms feel pulled up toward the ceiling, and their stomachs seem to "go up" also! As the elevator accelerates to a stop at the bottom, the opposite sensations are experienced by the passengers. Let us see if we can apply some of the physical principles that we have developed so far to learn more about these sensations.

Let us say that a 100-kilogram man is standing on a set of scales in the elevator. He assumes that the scale reading gives a measure of his weight. Think for a moment what a scale does. As the man steps on the scale, the spring inside compresses until it exerts an upward force on the man sufficient to hold him up. The pointer that the man reads *really measures the force that the scales are exerting up on him.* He *assumes* that the pointer on the scales gives him a measure of his weight. If the man is standing in a *stationary* elevator, or one moving at a constant velocity, this will be true. Let us use symbols to analyse first the situation with a stationary elevator (Case 1); then we will study the effect of an accelerating elevator on the scales (Case 2).

Case 1

Since $\vec{a} = 0$

$$\therefore \vec{F}_{net} = m\vec{a} = 0 \text{ (Newton's Law)}$$

But the forces acting on the man are

(1) the force of gravity
(2) the force of the scales

$$\therefore \vec{F}_{net} = \vec{F}_{gravity} + \vec{F}_{scales}$$

and $0 = \vec{F}_{gravity} + \vec{F}_{scales}$

or $$0 = m\vec{a}_g + \vec{F}_{scales}$$

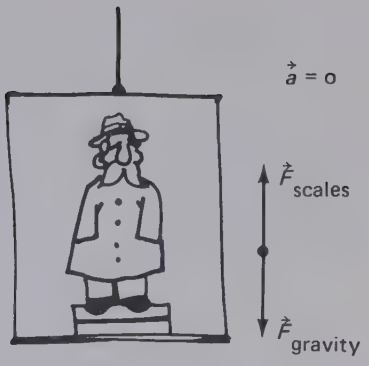
If we let all vector quantities directed downward be positive

$$0 = (100 \text{ kg} \times 9.8 \tfrac{\text{m}}{\text{s}^2}) + \vec{F}_{scales}$$

$$0 = 980\text{N} + \vec{F}_{scales}$$

and $$\vec{F}_{scales} = -980 \text{ newtons}$$

that is, the scales exert an upward force of 980 newtons on the man. This value is equal to the weight of the man. The force of the scales acts in the opposite direction to the force of gravity on the man, but the scale reading does not indicate direction, so it is of no consequence.



Case 2

Suppose that the elevator accelerates downward at the rate of 3 metres per second per second. Then, again using the convention that downward vectors are positive in sign,

$$\vec{a} = +3 \text{ m/s}^2$$

$$\vec{F}_{net} = m\vec{a} = +300\text{N}$$

but, as before,

$$\vec{F}_{net} = \vec{F}_{gravity} + \vec{F}_{scales}$$

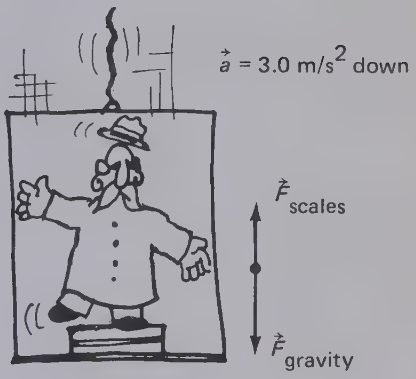
$$+300\text{N} = +980\text{N} + \vec{F}_{scales}$$

$$\vec{F}_{scales} = (300 - 980)\text{N}$$

$$\vec{F}_{scales} = -680 \text{ newtons}$$

The negative sign tells us this force acts *upward*.

Now the man reads his weight on the scales as only 680 newtons! He does not *actually* lose weight since the value of the force of gravity on him is still 980 newtons. But he *feels* lighter. Even the scale reading confirms his "lightness".



Q28 How does the reasoning for Case 1 have to change (if at all) if the elevator moves at constant velocity?

Q29 What will the scales read if the elevator accelerates downward at 9.8 metres per second per second?

See problems 5.23 to 5.25 on page 93.

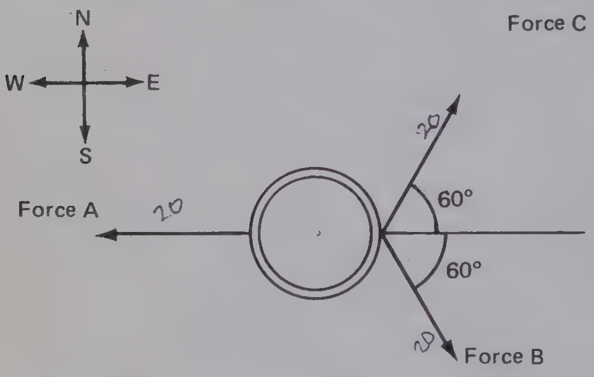
Problems Chapter 5

In these problems, use $\vec{a}_{gravity} = 10\ \frac{m}{s^2}$

Section A

5.1 Three boys each exert a force of 5 units due north on a car. A force of 6 units acts due south. Determine the magnitude and direction of the net force on the car:
a) by a scale vector diagram
b) by using a sign convention.

5.2 Three people each exert a force of 20 units on a large iron ring. Assume these are the only three forces acting on the ring. The forces act in the directions marked on the diagram.



a) Make a large scale diagram to determine the net force on the ring.
b) Could a sign convention be used in this problem to determine the net force on the ring? Discuss.
c) Suppose the person applying Force B lets go suddenly, but the others pull as before. Then what would be the magnitude and direction of the net force on the ring?

5.3 Determine the net force on a body which has a force $\vec{F}_1$ of 5 units acting on it at 40° to the north of west and another force $\vec{F}_2$ of 10 units acting due west.

* 5.4 Discuss the following questions first as an Aristotelian student, and then as a student of Newtonian physics.
a) Can a body be at rest if there are forces acting on it?
b) Can a body be moving with no forces acting upon it?

* 5.5 Explain the following with reference to the Law of Inertia.
a) A car driver suffers a "whiplash" injury when his stopped car is hit from behind by another car.
b) A ball released from the top of the mast of a moving ship lands at the base of the mast.
c) It is unsafe to approach a curve on a wet road at high speed.
d) Stamping your feet can remove snow from your boots.

* 5.6 A policeman arrived at the scene of an accident and found a load of lumber had fallen out the back of a truck onto the hood of a car behind. The truck driver claimed he was stopped at an intersection when the car rammed into his truck from behind. The car driver claimed that he stopped behind the truck and then the truck rolled back into him.
Could the Law of Inertia help decide which driver was correct? Discuss.

5.7 A force of 7.0 N is applied to a 2.0 kg cart. If the cart accelerates at 3.0 $\frac{m}{s^2}$, what is the value of the force of friction?

5.8 A man pushes a 5 kg box along a rough horizontal surface with a force of 30 N. A force of friction of 5 N opposes the motion.
a) What is the net force on the box?
b) What acceleration will result?
c) How hard should he push to double this acceleration?

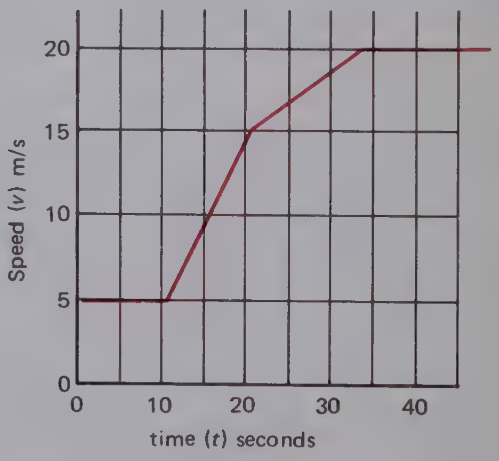
* 5.9 If it were possible to put a very massive object (Toronto's Commerce Court building, for example), on a frictionless surface, remove all the air surrounding it, and give it a shove, should it move? Explain.

* 5.10 A phrase often heard by police officers who stop speeding motorists on the highway is, "I didn't feel as though I was going that fast". Justify this statement on the basis of physical principles.

5.11 A cart accelerates from rest to 6 $\frac{m}{s}$ in 2 seconds. The force of friction is 5 N. If the cart is being pushed by an 11 N force, what is the mass of the cart?

5.12 A 1500 kg car moving at 15 metres per second crashes into a wall and comes to a stop in 0.5 seconds. Assume that the acceleration of the car while stopping was uniform. Calculate the collision force in the crash.

* 5.13 Given that the speed-time graph shown below depicts the motion of a 12 kg object and that the object is travelling on a frictionless surface, draw the graph showing the applied force during the same time period.



* 5.14 Name the reaction force to each of the following forces. Specify the object on which the reaction force acts.
a) A man holds up a book with his hand
b) The earth exerts a gravitational force on the moon
c) The north pole of a magnet exerts an attractive force on the south pole of another magnet.
d) A leaf falling to the ground pushes down on molecules of air.

* 5.15 A man is stranded near the centre of a frozen pond. Imagine that the pond surface is perfectly frictionless.
a) Could he walk to shore? Why?
b) How would you suggest he could get to the shore?
c) We admit that the above problem is artificial since no such pond exists. You have probably seen pictures of astronauts in situations that are very similar to this in principle however. Discuss one such situation.

* 5.16 A fishing line will break if it is subjected to more than 2000 N force. Will it break:
a) If an object weighing 2000 N is suspended from it?
b) If it is tied to a wall and pulled with a force of 2000 N.
c) If two men one at each end, both pull on the line with a 2000 N force?

* 5.17 A car is travelling along a road at a constant speed of 60 miles per hour:
a) Make a sketch showing all the external forces acting on the car. Indicate any forces that are equal to any others in the sketch.
b) If the brakes were applied, in what direction would the tires exert a force on the road? What force causes the car to stop?

* 5.18
a) What is your mass expressed in kg? (1kg = 2.2 lbs)
b) What is your weight in newtons?

5.19 Suppose that you want to experience a force of one newton pushing down on your hand. What mass should you place there?

* 5.20 If all air could be removed from their path, which would fall faster, a 10 kg steel ball or a 1 g feather? Explain why.

* 5.21 A boy weighing 500 N standing on a bathroom scale jumps upward:
a) Explain why the pointer on the scale will point to more than 500 N the moment that he jumps upward.
b) What will happen to the pointer when he lands? Explain why.

5.22 a) A frictionless air puck with a mass of 10 kg is pulled on a level table with a force of 2 N. What would its acceleration be?
b) Suppose that the same experiment were done on the moon. What would the acceleration of the air puck be? Why?

5.23 An object of mass 2 kg is held on a string. What is the tension in the string (ie: force exerted by the string on object) IF:
a) the object is at rest?
b) the object is moving downward at $2 \frac{m}{s}$?
c) the object is accelerating downward at $2 \frac{m}{s^2}$?
d) the object is accelerating upward at $2 \frac{m}{s^2}$?

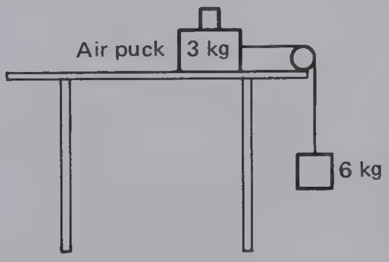
* 5.24 An astronaut on a journey to a nearby planet is part way between Earth and Mars, and is experiencing the feeling of weightlessness. He tries to distinguish between two identical looking packages, one of which weighs twice as much as the other on earth, but both appear weightless in his space-ship. Can he distinguish between them? How?

5.25 A 75 kg man stands in an elevator.
i) What force would the elevator exert on him, when:
a) the elevator starts moving upward with an acceleration of $1.5 \frac{m}{s^2}$?
b) the elevator is moving upward with a constant speed of $2.0 \frac{m}{s}$?
c) the elevator starts to accelerate downward at the rate of $1.5 \frac{m}{s^2}$?
ii) If the man were standing on a bathroom (spring) scale during his ride, what readings would the scale have in a), b), and c) above?
iii) It is sometimes said that the "apparent weight" changes when the elevator accelerates. What does this mean? Does the weight really change?

Section B

5.26 Two blocks touching each other rest on a level, frictionless surface. Block A has a mass of 10 kg, while block B has a mass of 6 kg. A force of 12 N is applied to block A in order to push the two blocks forward.
a) What is the acceleration of the "system" of two blocks?
b) What force does block A exert on block B?
c) What is the net force acting on block A?

5.27 A 3 kg air puck is attached by a string to a 6 kg block. The string is led over a pulley and hung over the edge of a table. What would be the acceleration of the system of two blocks when they are released? (Assume no friction between the pulley and string).



5.28 A sled has a mass of 4440 kg and is propelled by a solid propellant rocket motor of 890,000 N thrust which burns for 3.9 seconds.

a) What is the sled's average acceleration and maximum speed?

b) The data source states that this sled has a maximum acceleration of $30 \times a_g$. How can that be, considering the data given?

c) If the sled travels a distance of 1530 m while attaining a top speed of 860 m/s (how did it attain *that* high a speed?!), what is its average acceleration?

* 5.29 Consider a tractor pulling a heavy log in a straight line. On the basis of Newton's third law, one might argue that the log pulls back on the tractor just as strongly as the tractor pulls the log. But why, then, does the tractor move?

* 5.30 Discuss the statement: "There is not a single natural phenomenon which could be discovered by logic alone".

* 5.31 The great philosopher and mathematician, Bertrand Russell once said that Aristotle "has been one of the great misfortunes of the human race". In the light of our references to his work in this unit, assess this statement.

Epilogue

We have seen in this Unit that Aristotelian physics, which provided acceptable explanations of motion and its causes for two thousand years, was replaced by the physics developed by Galileo and Newton. This new system had completely different answers to the same questions. We have seen that the writings of Aristotle came in time to be regarded as irrefutable and beyond reproach. His views became so ingrained in the minds of people that they were not open to question or doubt.

By presenting the main features of Aristotelian physics and asking you to try to understand some of the reasons why it was accepted, we hope that now you will view the formulae of Newtonian physics not so much as being true, but in a rather more open-minded and perhaps critical light.

The challenge presented by Galileo and Newton was one of rethinking a traditional set of fundamental ideas and of accepting a conclusion that was totally contrary to these. Such exercises are extremely exciting and important because they emphasize the need to continually examine the "common sense" we inherit from earlier ages. The history of physics, as with nearly all fields of human thought, provides many examples. Around 1900, for example, some important principles of Newtonian physics were questioned.

According to Newton:

> "Absolute space in its own nature and without regard to anything external, always remains similar and immovable."
>
> and
>
> "Absolute, true and mathematical time, of itself and by its own nature, flows uniformly on, without regard for anything external."

In his "Special Theory of Relativity", published in 1905, Albert Einstein predicted that these concepts of "absolute space" and "absolute time" were faulty. Einstein wrote that neither measurements of length (an interval of space) nor time are "absolute" and "without regard to anything external" but *depend on the speed of the object being measured relative to the speed of the observer* doing the measurement. Einstein claimed:

1. The length of an object as perceived by an observer in motion with respect to the object, is shorter than the length of the same object, as perceived by an observer at rest with respect to the object.
2. The time elapsed during an event actually depends on the state of motion of the observer.

These claims strike at the very heart of physics, for one can scarcely make any statement in physics without direct or indirect reference to the terms length and time. Speed, acceleration, force, energy, momentum, all depend on these fundamental concepts. The British Astronomer, Arthur Eddington wrote in 1922:

> Surely then we can best indicate the revolutionary consequences of (relativity) by the statement that distance and duration, and all the physical quantities derived from them, do not as hitherto supposed refer to anything absolute in the external world, but are relative quantities which alter when we pass from one observer to another with different motion. The consequence in physics of the discovery that a yard is not an absolute chunk of space, and that what is a yard for one observer may be eighteen inches for another observer, may be compared with the consequences in economics of the discovery that a pound sterling is not an absolute quantity of wealth, and in certain circumstances may "really" be seven and sixpence.

Although many physicists became convinced of the validity of Einstein's theoretical predictions solely on the basis of reading his "paper", others were slower to accept Einstein's contentions, and some scientists rejected the entire theory outright. As historian Ronald Clarke puts it, "the concept of relativity percolated slowly through the accepted ideas like rain through limestone rather than breaking them down like the weight of water cracking a dam". Einstein realized that full acceptance of a theory in science never results until there is considerable experimental evidence in support of the theory. He wrote "It will be possible to decide whether or not the foundations of relativity theory correspond with the facts only if a great variety of observations is at hand". Such observations have been accumulating rapidly. The special theory has now gained universal acceptance because of widespread evidence from many independent experiments and observations. We shall recount here just one of the many observations which support Einstein's predictions.

Certain tiny particles, called muons, are produced at speeds very close to the speed of light high in the earth's upper atmosphere, about 60 kilometres above the surface. Muons are particles that have very short (by our time scale) lifetimes. In any collection of a large number of muons, half of them will have "decayed" into different particles in about 2 microseconds. Any muon that decays in this time would only travel a distance of about 600 metres before ceasing to be a muon. Very few muons should "live" long enough to travel the 60 km from where they are produced to the earth's surface.

However, the observed facts do not fit this statement. Many times more muons are detectable at the earth's surface than could be expected from the reasoning outlined above. How can we account for the fact that so many muons arrive at the earth?

An answer can be found in Einstein's theory

To an observer on earth, time, as measured on a "clock" that we might imagine travelling along with the muon, runs about 100 times slower than earth time. Thus, the speeding muon's lifetime, as seen by an earth observer, is 100 times longer than 2 microseconds (the lifetime of muons that are not moving relative to the observer). In this time, the muon could travel 100 times farther than it could in 2 microseconds. That is, instead of travelling 600 metres, it could travel the 60,000 metres. (60 kilometres) required to get from the point where it was produced to the earth's surface.

Scientists now think that if people were able to travel at this speed relative to the earth, then the travellers' lifetimes, as perceived by observers on earth, would be 100 times greater than their present, earthbound, lifespans. However, if a human passenger were carried on a muon, *he* would not be able to detect any difference in the flow of time of his clock. His entire biological system, the workings of his heart, muscles, cellular and molecular processes, plus any clock or watch he might have, would all slow down again relative to our earth-observer by the same amount. Time, to the traveller, would be the same as it always was.

Ah, but how would he account for the fact that the muon "made it" to earth over the distance of 60 kilometres without decaying? Again, Einstein's theory offers an answer. To him, the distance of travel is shorter than it seems to us on earth. At his speed of travel, lengths at rest with respect to the earth would appear 100 times shorter than they appear to us on earth. A height of 60 kilometres, as measured by an earth observer, would appear to be only 600 metres to the muon rider!

A new perspective to the claims of special relativity was given in 1908 by the German mathematician Minkowski. He formulated mathematical expressions involving the ideas of Einstein's relativity which blended space and time into a single co-ordinate system. In an address delivered in 1908, Minkowski began:

The views of space and time which I lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth, space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.

Out of this difficult to perceive concept of space-time has come another realization, also predicted by Einstein. This prediction involves our concept of mass. If one body is in motion with respect to an observer, the *mass* of the body in motion, as measured by the observer will be larger than the mass it would have when it is at rest with respect to the observer. Einstein predicted that this increased mass could be quantitatively calculated by the relation

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where m_o = the mass of a body at rest (called its rest mass)
v = the speed of the body relative to an observer
c = the speed of light
m = the mass of the body relative to the observer (called the relativistic mass)

putting values of the speed of a body into this relationship shows that the relativistic mass m, differs measurably from the rest mass (m_o) only at extremely high speeds. At the maximum speed obtained by a rocket ship on its trip back to earth from the moon, about 10^4 m/s, a one kilogram object would increase in mass by less than .0000001%. Particles of size comparable to atomic nuclei, however, are often given speeds within a few percent of the speed of light, and their increase in mass over their rest mass has been measured and confirmed to be exactly in accordance with that predicted by Einstein! Machines that accelerate electrons and protons to within a thousandth of a percent of the speed of light must be designed to allow for this increase in the mass of the particles with speed. The increase in the mass of bodies travelling at speeds close to the speed of light also suggest that no body can be accelerated to a speed beyond that of light. As the speed of a body becomes very close to that of light, the mass approaches an infinitely large value. An infinitely large force would be required to accelerate the body further. Since there are no infinite forces, a body cannot be accelerated to a speed greater than that of light!

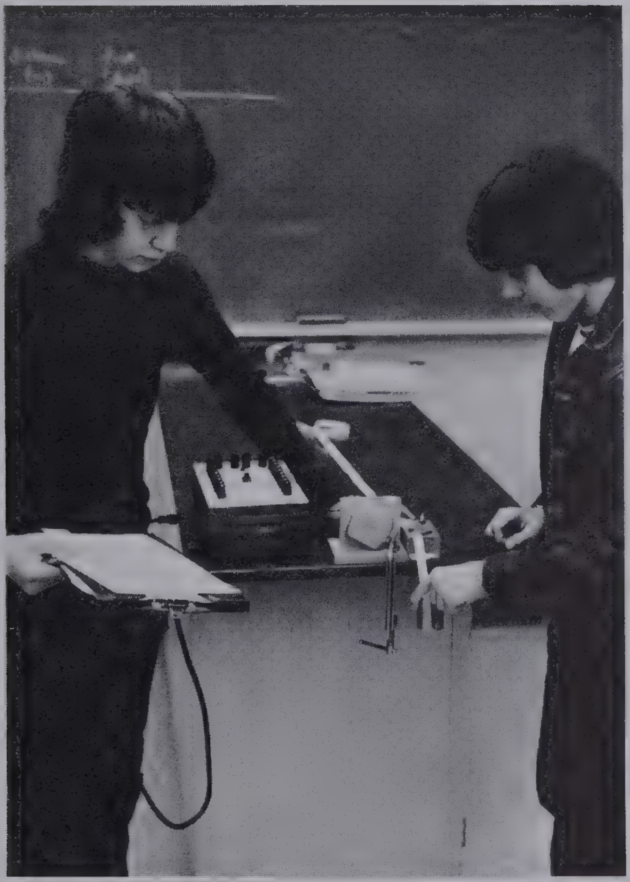
In 1920, it was argued that in the coming generations, people would be schooled in the concepts of relativity and would come to accept its consequences as everyday matters quite in harmony with common sense. Einstein, having heard so often that his ideas violate common sense, said "common sense is a body of prejudice laid down in the mind prior to the age of 18". Today, 70 years after the publication of the special theory of relativity, this widespread knowledge and acceptance of the theory has not occurred. Most people still conceive of space and time in the absolute, Newtonian sense. This is interesting, for it is not so much that the propositions of relativity are so difficult to understand, it is that they are so difficult to believe. Perhaps this is essentially the same kind of problem as that faced by Galileo as he demonstrated the facts of freely falling bodies to audiences that were not so much unable to see as they were unwilling to believe.

As for the future, physicist Kenneth Ford points out that . . .

> The special theory of relativity has raised a host of questions which for the present must be called philosophical, although they may one day become scientific. Why, for example, does man perceive space and time in such different ways if they are really only parts of the same entity, space-time? Why, if we can move this way or that in space, do we find ourselves moving inflexibly in only one direction through time? Are there other parts of the universe or other eras in the history of the universe when the direction of motion through time was different? If we are three-dimensional creatures imbedded in a four-dimensional world, are there still more dimensions still further removed from our perception? (By 4-dimensional, we mean 3 spacial co-ordinates and one space-time co-ordinate.) Such questions, apparently idle today, give some indication of the kinds of questions that might be answerable within the framework of science tomorrow.

I

Handbook



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Preface

The purpose of this handbook is to provide you with some guidance in things which you can do in order to get some first-hand experience with the techniques of scientific investigation and the ideas and concepts of this physics course.

This handbook is different from many other such books in that it contains far more material than you would ever be expected to do in a single course. You will have to choose, with the help of your teacher, experiments and activities which will be the most interesting and useful to you.

For the most part, there is a handbook chapter related to each chapter of the text. In each chapter, there are usually three sub-sections: *Experiments, Activities,* and *Film Loop Notes.*

For each *Experiment,* instructions are outlined fully and may be followed by the entire class. In some cases, alternate procedures are outlined and you may choose one or more on the basis of available equipment or personal preference. At the end of some experiments are *Additional Questions* which may suggest further experimentation with which you could deepen your understanding, or develop additional techniques.

In the *Activities* are suggestions for mini-experiments, exercises, or projects which you can do on your own time. Detailed instructions for the activities are not usually given so that you may develop them by yourself in your own way.

Some film loops are recommended as part of this course. The *Film Loop Notes* provide background about these loops and give instructions for their use.

At the end of the handbook are several *Appendices.* Some explain techniques which will be useful in various experiments. For example, Appendix A provides instructions for the operation of a camera. Others contain useful reference information.

Keeping Records of Laboratory Work

An essential part of doing laboratory work is the recording of results as the experiment proceeds. It is essential, because later, you or someone else will probably want to know what happened. In this section is some advice on how to keep a good record of your laboratory work. Your teacher may suggest a particular format for your reports, but, regardless of the type of record to be kept, there are some general principles which you should always try to follow.

1. Make your report sufficiently clear and complete so that months after its writing you will be able to pick up your report and from it explain to yourself or someone else exactly what happened.

2. Keep a complete record *as you do the experiment!* It is a bad habit to put data on a scrap of paper and then recopy them into your notebook later.

3. As you do the experiment, include in your record, answers to questions which occur to you or are given in the handbook. They are often important to your understanding of what is happening in the experiment.

A complete record usually contains the following parts.

Aim. You should know what you are trying to do before you start the experiment and should write your aim at the top of your report before you begin.

Apparatus. A good way to record what you use is to prepare a sketch of the apparatus. Label the main parts. List the types of equipment and any special settings or adjustments in case you wish to repeat the experiment at a later time.

Procedure. Outline the main points of what was done. Reference to the handbook for a description of the procedure may be adequate in some cases. (Such a reference would save some time.)

Observations. Organize all numerical data, if possible, in tabular form. Always identify the units, (metres, kilograms, seconds, etc.) for all of the data you record. If you suspect that a particular piece of data is not very reliable, (perhaps you measured it too quickly or the apparatus shifted), cross it out and make an explanatory note of the fact. Do not erase the reading. You may find out later that it was correct after all and that you, not the data, were in error.

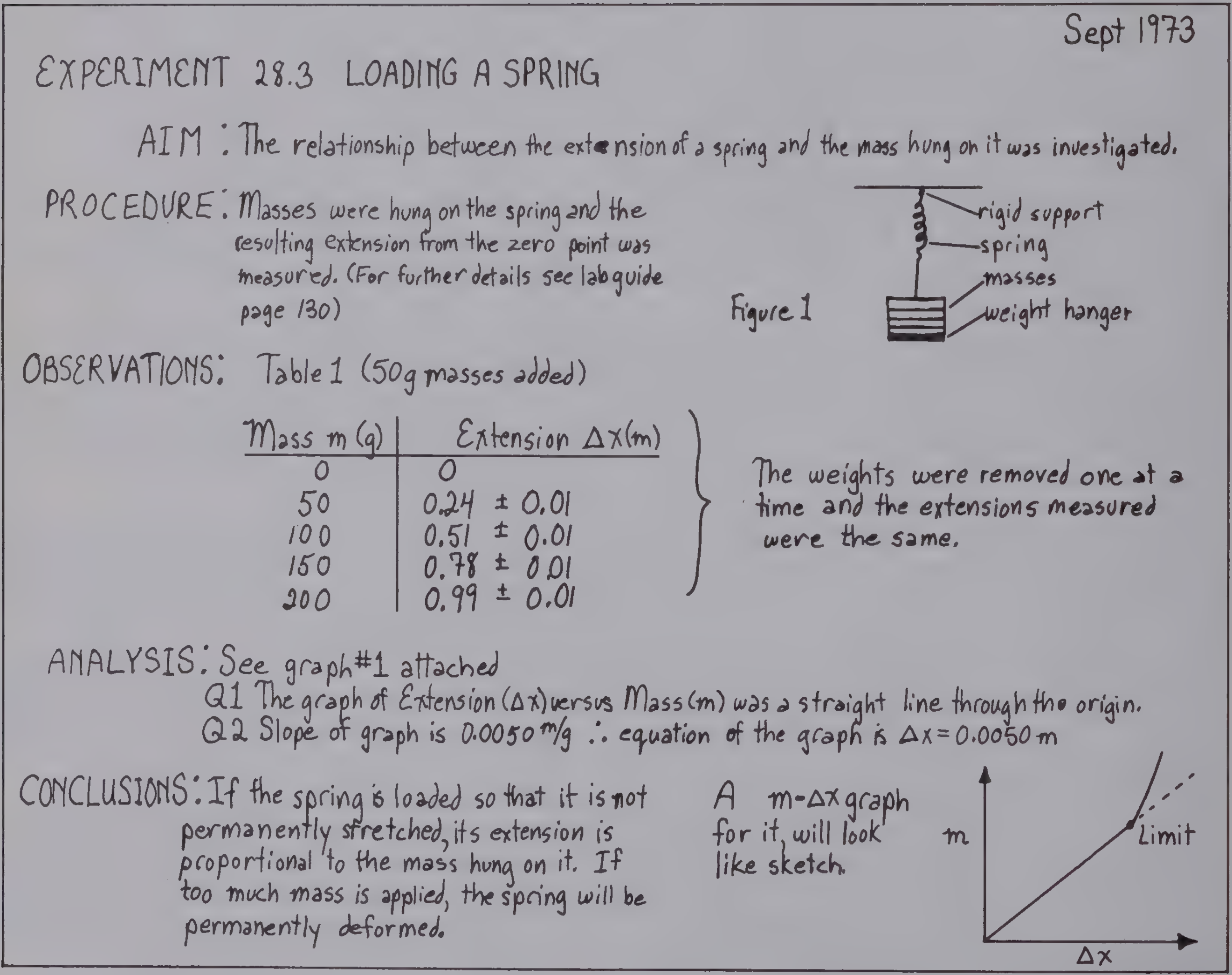
Analysis. Once you have collected the data you will interpret what they mean. Your analysis is the means by which you arrive at conclusions from your observations. While doing this analysis, do not ask, "What was supposed to happen?". Rather ask, "Based on my observation what must have happened?". In trying to answer this last question, show all steps of reasoning and calculation clearly.

A good graph often makes the meaning of your results much clearer. If you include graphs in your analysis, be sure that their form is correct and refer to their characteristics in drawing conclusions. Appendix B outlines a correct form and procedure for graphical analysis.

Include in the analysis, a discussion of sources of error or uncertainty and try to estimate how large each could be. If you want to take this calculation a step further, you could estimate the total uncertainty in your result due to the uncertainties in several separate factors. (Appendix C discusses measurement and uncertainty.)

Conclusions. *Conclusions must be based on your observations.* They should summarize what you learned from the experiment, not what you think "they" want (your teacher, the authors of the text . . . , or whoever). To relate conclusions to specific observations, a good format to use for a statement of a conclusion is, *since* . . . happened, *therefore* . . . must be true.

There is no "wrong" result in any experiment. If your observations and measurements are correct within the limitations of uncertainty, and your analysis is sound, then your results will be correct. Whatever happens in nature, including the laboratory, cannot be "wrong".



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EXPERIMENT 28.3 LOADING A SPRING

AIM : The relationship between the extension of a spring and the mass hung on it was investigated.

PROCEDURE : Masses were hung on the spring and the resulting extension from the zero point was measured. (For further details see lab guide page 130)

Figure 1

rigid support
spring
masses
weight hanger

OBSERVATIONS: Table 1 (50g masses added)

Mass m (g)	Extension Δx(m)
0	0
50	0.24 ± 0.01
100	0.51 ± 0.01
150	0.78 ± 0.01
200	0.99 ± 0.01

The weights were removed one at a time and the extensions measured were the same.

ANALYSIS: See graph #1 attached

Q1 The graph of Extension (Δx) versus Mass (m) was a straight line through the origin.

Q2 Slope of graph is 0.0050 m/g ∴ equation of the graph is $\Delta x = 0.0050\,m$

CONCLUSIONS: If the spring is loaded so that it is not permanently stretched, its extension is proportional to the mass hung on it. If too much mass is applied, the spring will be permanently deformed.

A m-Δx graph for it, will look like sketch.

Chapter 1. Physics Begins

Science attempts to understand the Universe. This process of understanding invariably begins with a question which may be raised because of some strange phenomena or unexplained chain of events. From such questions, the scientist gains his inspiration to investigate by collecting data and searching for patterns or relationships which will explain what has been seen. For example, early man was intrigued by what happened in the heavens. Events such as motions of the sun, moon, stars; eclipses; supernovae; lightning and so on, which we understand fairly well today were, at one time, strange puzzles. Scientists such as Ptolemy, Galileo, Newton, and Einstein have attempted to explain these events. In fact, Einstein described what the scientist seeks to understand as " . . this huge world that exists independently of us human beings . . . that stands before us like an eternal riddle".

Experiment 1.1 Some Puzzling Events

In the first experiment you will investigate several puzzling events and attempt to understand what is happening. Several devices are set up at stations around the room. Investigate at least four of them during the class period. If you already know about some of them, leave those and look at the others. For each station that you choose you should: (a) describe what happens, (b) answer the questions provided, and (c) try to explain what happens.

The explanation of some of these events will be far from obvious now. But as this course develops, you will master concepts which will make it possible for you to explain what you have seen.

Demonstration 1 Cloud Chamber

Look for evidence of motion in the chamber. You should be able to see vapour trails.

Q1 Where do the tracks come from?
Q2 Describe the tracks. Are they all the same? Discuss.
Q3 Why do the tracks disappear?

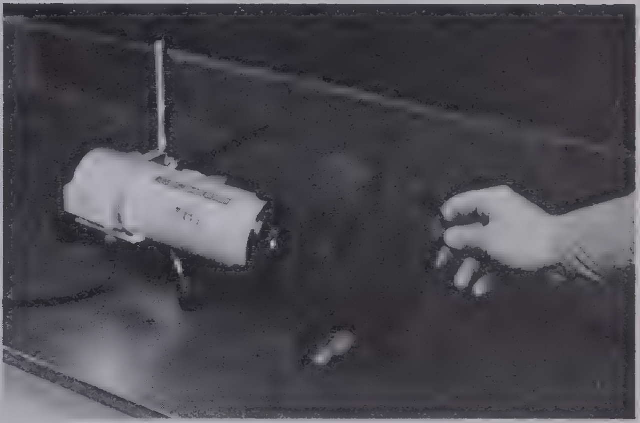
Figure 1.1

Demonstration 2 Polaroid and Light

Set up one sheet of polaroid at right angles to the path of the light beam. Now line up a second one and rotate it through 180 degrees keeping it parallel to the first one.

Q1 What happens to the light intensity transmitted through both sheets as one is rotated?
Q2 Does it matter which sheet is rotated?

Figure 1.2



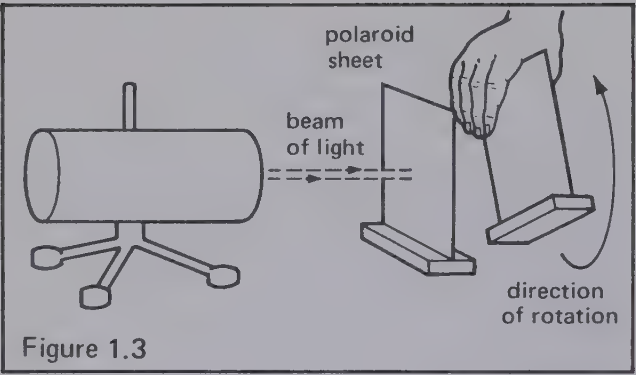


Figure 1.3

Now set up both sheets so that no light is transmitted through both. Place a third sheet between the other two and rotate it slowly, keeping it parallel to the other two sheets. Watch the intensity of transmitted light.

Q3 What happens? Why?
Q4 Under what condition is the amount of light transmitted by all three sheets, maximum?

Demonstration 3 Variable Air Column

Sound a 512 Hz tuning fork by hitting it with a rubber hammer. Place the fork about 2cm above the top of the air column. Vary the column's length by moving the glass tube and the fork up and down together, *slowly*.

Q1 How is sound produced?
Q2 Which is louder, the fork alone or the fork above an air column adjusted for maximum loudness? Why?
Q3 How many column lengths for maximum loudness can you find? Try forks of different pitch.
Q4 How does a slide trombone work?

Figure 1.4

Demonstration 4 Fan Cart

First, turn on the fan so that it blows on the sail attached to the cart. Then remove the sail but leave the fan still blowing.

Q1 What happens in each case? Why?
Q2 What change would you expect in each case if the propellor were to turn faster?
Q3 How could the fan be used to propel the cart by blowing on the sail?

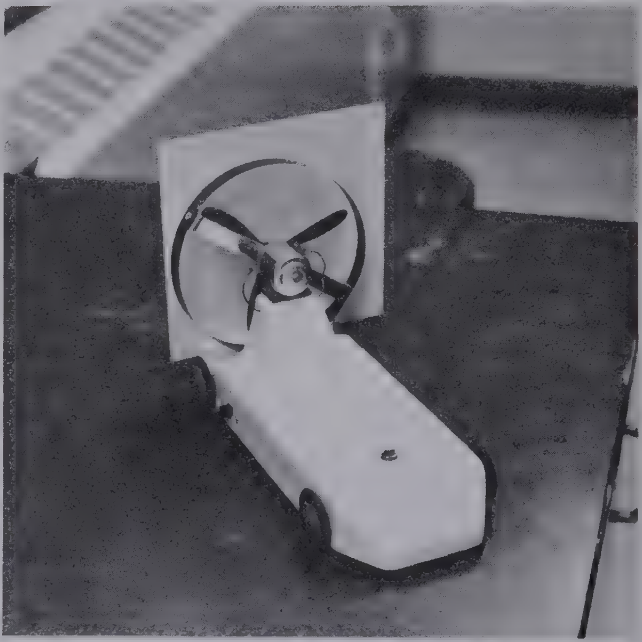


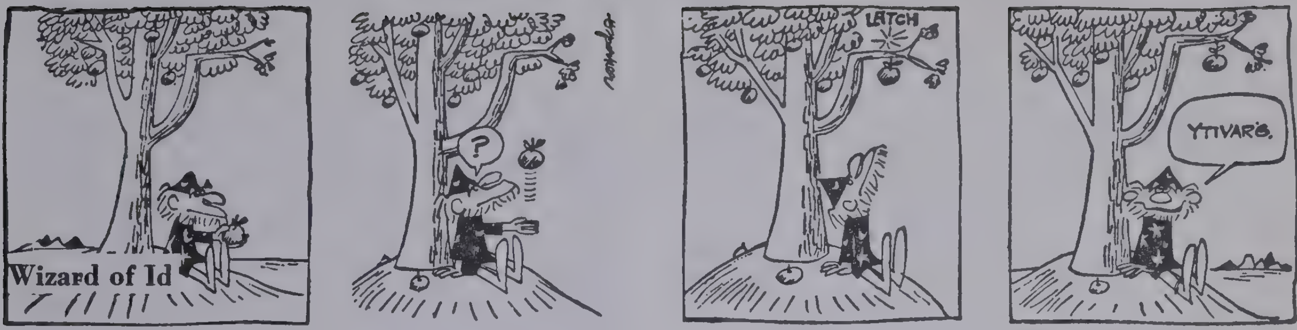
Figure 1.5

Demonstration 5 Thought-Problem

Sometimes an investigation can be carried out, not by performing an experiment with an actual object, but by thinking about it. Most real experiments begin this way. Here is an example.

Consider throwing a ball straight up above the earth.

Q1 What happens to its speed during its flight?
Q2 Does it ever stop and if so for how long?
Q3 Why does it fall back down?
Q4 Can you state a general rule about all objects above the earth's surface?
Q5 Why does the moon not fall to the earth?
Q6 Why does the sun not fall to the earth?



Demonstration 6 Action – Reaction Toy

Pull one of the spheres aside and then release it so that it strikes the others. Repeat pulling two, then three, etc., aside and releasing them.

Q1 What is a general rule which can be used to predict what will happen in each case?
Q2 The spheres swing from side to side after they are started. Will this motion continue forever? Discuss.

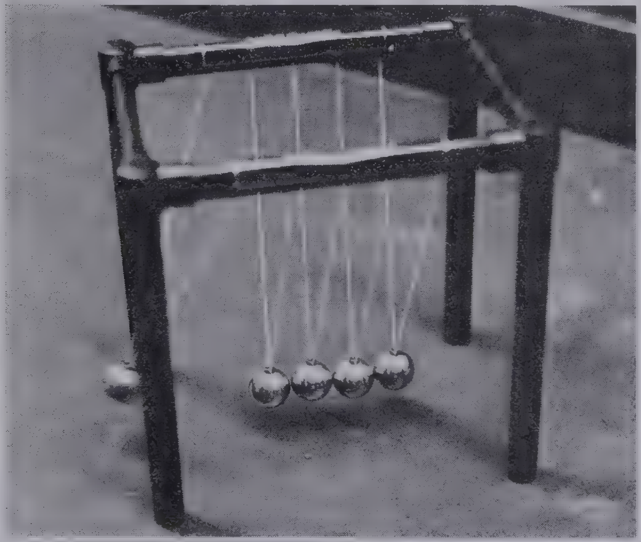


Figure 1.6

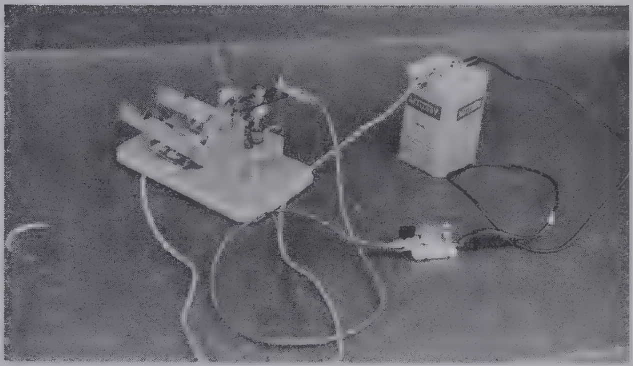


Figure 1.7

Demonstration 7 St. Louis Motor

Close the switch and then give the motor's rotor a small push to get it started, if necessary.

Q1 What will happen if the terminals of the battery are reversed? Try it.
Q2 Will the motor work if
(a) the poles of both magnets are reversed?
(b) the pole of one magnet is reversed?
(c) the magnets are removed?
Q3 What is the purpose of
(a) the battery?
(b) the magnets?

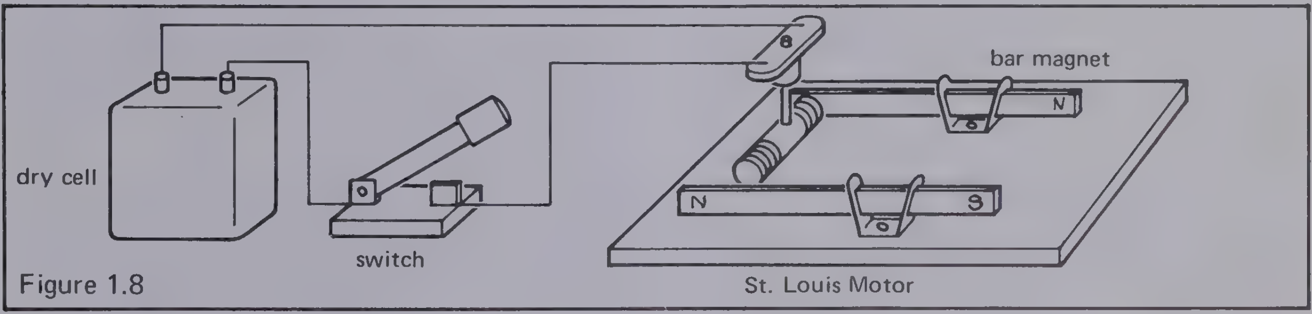


Figure 1.8

Experiment 1.2 Falling Objects

No doubt the Greeks experimented with falling objects because the natural motion of falling objects was very important in Aristotle's theory of motion. However, they did not have accurate devices for measuring short time-intervals, so in doing their experiments, they probably made a judgement rather than a measurement. A stone falling through the air toward the earth in the way Aristotle predicted, moves so rapidly that it is difficult to make a good judgement about its motion. If the stone falls in a heavier medium such as water, it is much easier to judge its motion. In this experiment you will investigate the motion of objects falling in water and in other media, without making precise measurements, just as the Greek men of science probably did.

Procedure

Set up a tall cylinder of water as shown in Figure 1.9. Place a wad of paper towels or a sponge in the bottom of the cylinder as a cushion. Obtain several marbles and steel spheres of various sizes. The steel spheres can be lifted from the bottom of the cylinder with a magnet. (This experiment can also be done in a swimming pool where the distance fallen will be much greater.)

Drop a marble from just below the surface of the water and watch how quickly it falls from the top to the bottom. Record your observations.

Repeat with a steel sphere of the same size and in each instance record what you see.

Then, drop the marble through the same distance in air and compare results with a similar object in water. If a thicker medium such as oil is available, repeat the drop and in each case record your observations.

Analysis

Try to keep an open mind as you analyse your results. Use your observations to answer the following questions.

Q1 How did the speed of the object behave as it fell through a medium?
Q2 What effect did the object's weight have on its rate of fall?

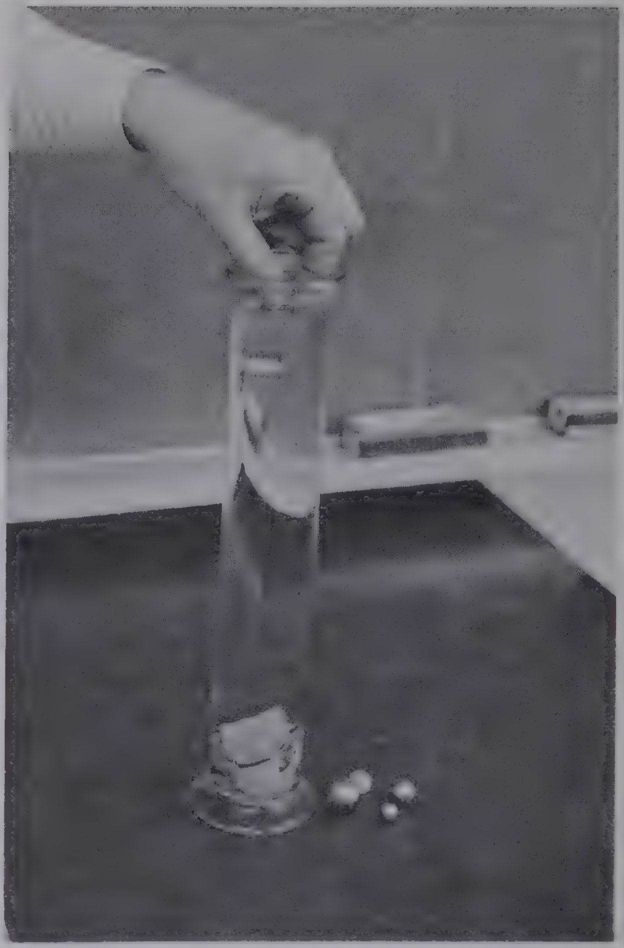


Figure 1.9

Q3 What differences in the object's motion were caused by different media? Why?
Q4 How do your conclusions compare with those of Aristotle? Discuss differences and similarities.
Q5 Based on your conclusions for an object falling in water, what can you predict about the behaviour of objects falling in air?
Q6 What do you think is the major difference between the ways in which men of modern science and of Greek science investigate nature in order to form theories.

Chapter 2. From the Complex to the Simple

Uniform motion is the simplest type of motion and yet it is the most difficult to attain. In the first experiment you will study uniform motion and yet you will not really see it. Instead, you will try to come as close as possible to achieving this idealized type of motion. As the experiment proceeds you will measure the distance travelled and time elapsed and analyse your data to see how close to uniform the motion was.

Experiment 2.1 Uniform Motion

There are three procedures outlined. You may choose any one of them.

A Record the object's position on a ticker tape pulled through a vibrating timer by the object.

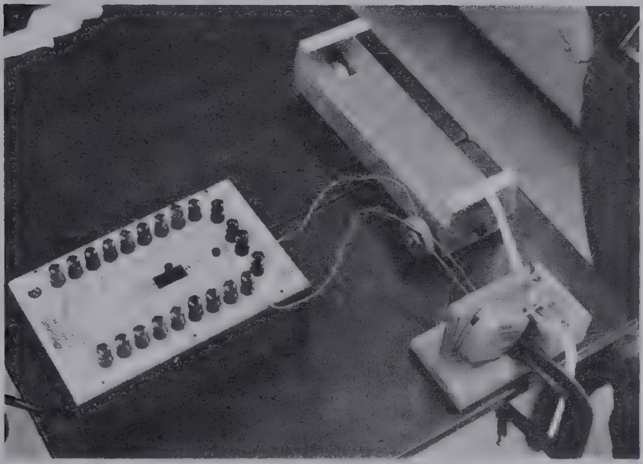
B Photograph the object in motion with its position at equal time-intervals indicated by a stroboscope or blinky.

C Measure, with stopwatches, the times required for the object to travel a series of equal distance-intervals.

Procedure A Ticker Tape and Timer

Set up the ticker tape and timer on a horizontal table as shown in Figure 2.1.

Figure 2.1



Attach approximately one metre of ticker tape to the cart and thread it through the timer under the carbon disc.

Start the timer and pull the cart at a constant speed in a straight line across the table. Stop the timer and repeat with a second tape at a different constant speed so that you and your partner each have one tape to analyse.

You have enough data now if you are willing to measure time in terms of the period of your timer. But if you want to know the time-intervals in seconds, you will need to calibrate your timer using a stopwatch.

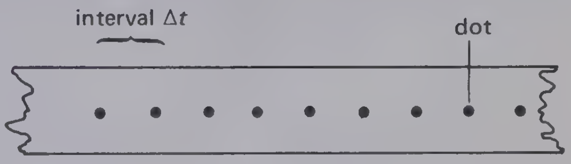


Figure 2.2

CALIBRATING THE TIMER

To calibrate the timer, take another piece of tape approximately one metre long and thread it through the timer. Start the timer. Using a stopwatch in one hand, and with the tape in your other hand, begin pulling the tape and start the watch at the same time. Pull the tape for exactly one second and then stop. To find the length of the time-interval between dots, Δt, we must assume that the timer vibrates at a constant rate. If it does, then the time-interval Δt, in seconds, is given by

$$\Delta t = \frac{1 \text{ second}}{\text{number of intervals in one second}}.$$

Note: As shown in Figure 2.2 while the timer makes 5 dots it makes only 4 intervals.

MEASURING THE TAPE

Disregard the non-uniform motion of starting and stopping and pick as a starting point the first dot from where the motion appears uniform. Mark this dot t=0. To limit the number of intervals mark every fifth dot, and use as the time-interval, the time taken to make these five dots. You can call this time-interval, *1 tock* (an arbitrary unit). If you have calibrated your timer, you can convert this unit to seconds. Your tape should look something like the sample segment shown in Figure 2.3.

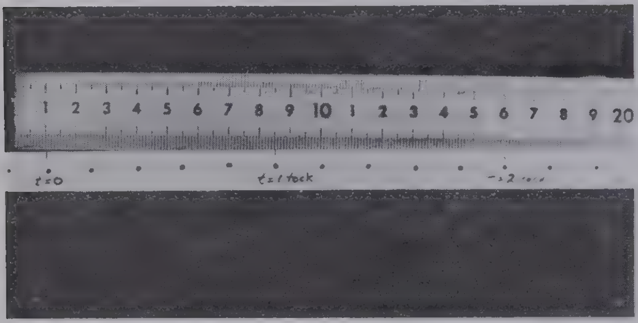


Figure 2.3

Measure to the nearest millimetre the distance from the start to the position of the object at the end of each successive time-interval (x_1, x_2 x_3 . . . as shown in Figure 2.4). Tabulate your measurements in a table with headings similar to Table 2.1.

Units for time can be either your arbitrary units (e.g. *1 tock = time to make 5 dots)* or seconds, if you have calibrated the timer.

Table 2.1

Time t (tock)	Position x (cm)	Distance travelled in the interval Δx (cm)	Average speed v in the interval ( )
0	0		
		2.0	
1	2.0		
		1.9	
2	3.9		
3			
⋮	⋮	⋮	⋮

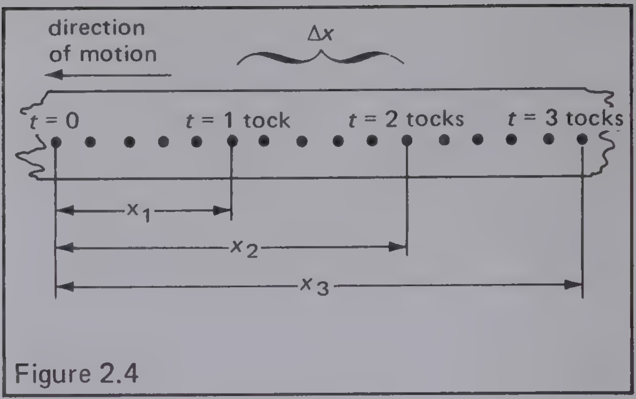


Figure 2.4

Procedure B Photograph

Another method of obtaining nearly uniform motion is to roll or slide an object on a surface where there is as little friction as possible. There are several ways to try this. Three possible arrangements are shown in the Figures 2.5, 2.6, and 2.7. You may be able to design another method which also works well.

Figure 2.5

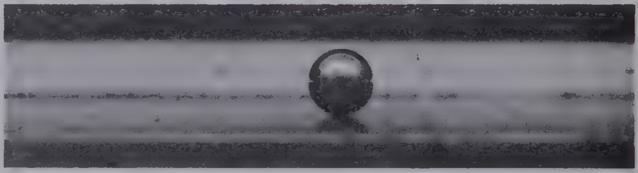


Figure 2.6

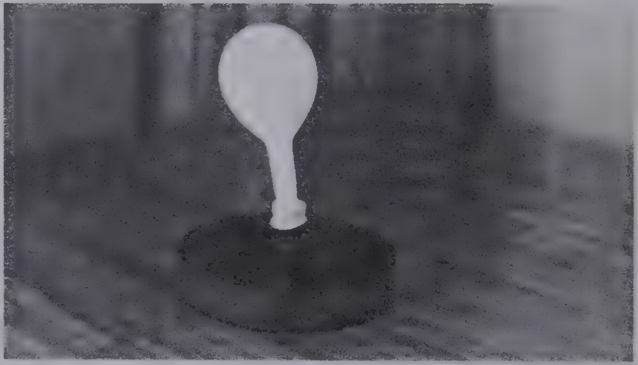
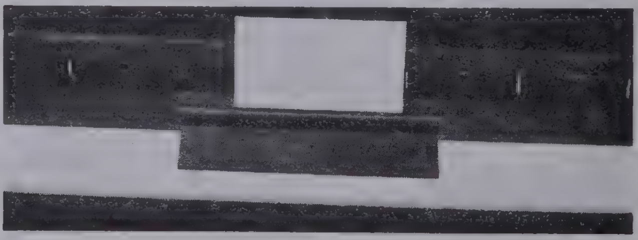


Figure 2.7



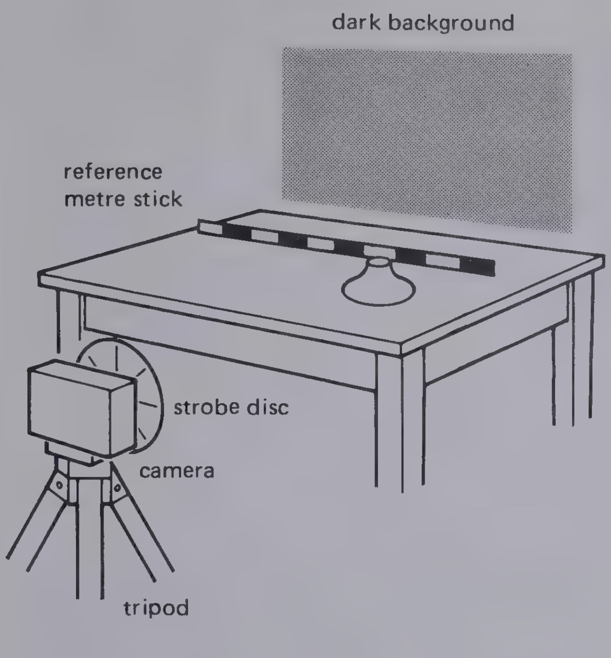


Figure 2.8

A time-exposure photograph taken through the slits of a rotating mechanical stroboscope or with the light of a flashing electronic stroboscope in a darkened room, will show the object's position at equal time-intervals. Or, you can achieve the same result using a blinky light (attached to the object), blinking at a constant frequency.

Set up the camera on the same level as the object and about 1 to 2 metres away. One of you can operate the camera while the other launches the object. (Detailed suggestions for operating the camera are given in Appendix A.) Adjust the strobe or blinky frequency and the speed of the object so that you get at least five images on the photo. Place a reference metre stick on the table parallel to the intended path of the object so that later you can determine a scale for your measurements. Then, try a dry run before taking the first picture. Next, take two good pictures of the object launched at two different speeds so that each of you has one.

Measure the distance from the starting point to each image. (See Appendix A for suggestions on measurement.) Record the distances and the times in a table similar to Table 2.1.

Procedure C Stopwatches

Mark off at least 5 equal distance-intervals as far apart as possible along the object's path. (You can use a Hot Wheels car or one of the objects shown in Figures 2.5, 2.6, or 2.7 travelling along a smooth, level track.)

Place one observer with a stopwatch at each marker. Each observer should start his watch when the car is launched and stop it when the object passes his marker. Repeat at different speeds until every member of the team has a separate set of data.

Record the distance from the starting point to each marker and the corresponding times in a table similar to Table 2.1.

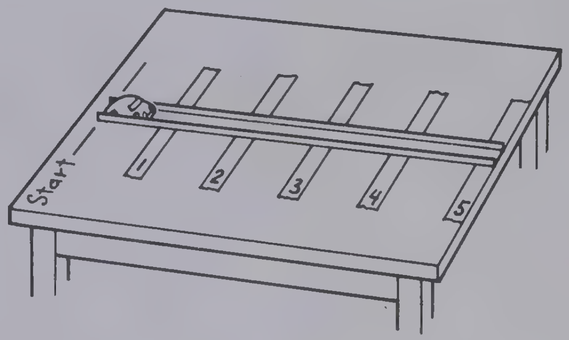


Figure 2.9

Analysis

Q1 From your measurements, does the motion appear uniform? Discuss reasons why it may not appear exactly uniform.

Q2 Why is it not necessary to measure time in seconds in order to answer Q1?

Plot a graph with distance on the vertical axis and time on the horizontal axis.

Q3 What is the shape of the line of "best fit" for the distance-time graph?

Repeat the analysis for the second motion at a different speed and plot the distance-time graph for it on the same axes as you used for the first one.

Q4 How do the two graphs compare?

Q5 What characteristic of the distance-time graph is determined by the speed of the motion?

If you have not already done so, calculate the distance travelled, Δx, for each time-interval in your table. Then calculate the average speed for each interval using the formula $v_{av}=\Delta x/\Delta t$. Be sure to include units. Also calculate the average speed for the entire trip.

Find the slope of the distance-time graph for each run. Remember that the slope will have units. Be sure to show them.

Q6 How should the slope of each graph compare with the average speed for the run? Why? Compare them.

Plot a graph of speed (on the vertical axis) versus time (on the horizontal axis). The speed values in your table represent the average speed for each time-interval. *If the interval is fairly small, the average speed will be a good estimate of the actual speed at the mid-point of the time-interval.* Consequently, the speed value for the first interval should be plotted at t=0.5 tocks, for the second at t=1.5 tocks, and so on as shown in Figure 2.10.

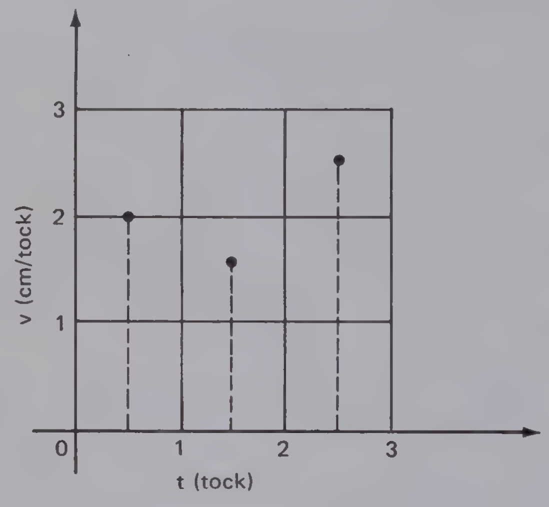


Figure 2.10

Q7 What is the shape of the speed-time graph for uniform motion?

Calculate the total area under the speed-time graph. Show units.

Q8 How does the area under the graph compare with the actual distance travelled in that time?

Additional Questions

Q9 What would the distance-time and speed-time graphs look like if the motion were not uniform? Sketch these graphs for a car

(a) steadily gaining speed.

(b) slowing up on spotting a radar trap.

Q10 As we said earlier in the experiment, uniform motion is difficult to obtain. Also, in making measurements, there is always some uncertainty. Try to estimate the uncertainty in your measurements for the experiment. (Appendix C gives some basic rules to help you with this.) Were the differences in the speed from interval to interval within the limits of the uncertainty of your measurements?

Q11 Where would you expect to see perfect uniform motion of an object which is not powered? Discuss.

Experiment 2.2 Accelerated Motion

Here is a stroboscope photograph of the motion of a ball. The strobe was flashing at a constant rate of 10 flashes per second.

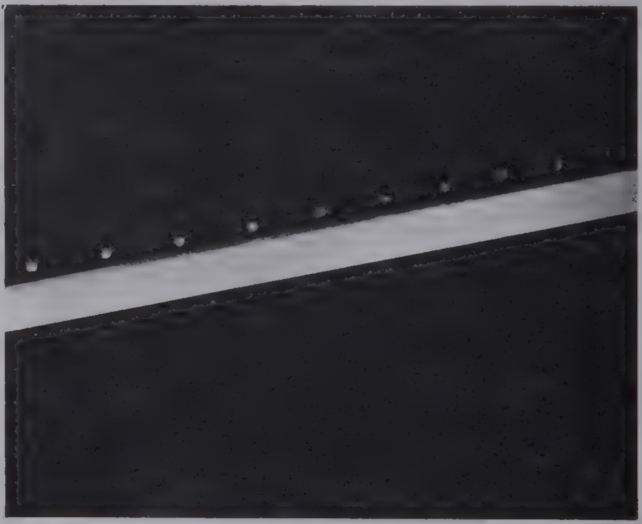


Figure 2.11

Is the motion uniform? How can you tell?

In the previous experiment, it became clear that uniform motion occurs only under ideal circumstances. All other motion is non-uniform. What new concepts are necessary to describe non-uniform motion? You can use the same definitions of distance travelled and average speed, but the behaviour of these quantities with respect to time will be different. Also, you can obtain a record of the motion in the same way as in Experiment 2.1 by finding the position of the object at equal time-intervals using a camera or a recording timer. The analysis can be extended to include acceleration.

An alternative method is to set up position markers along the path of the object and measure how much time it takes to travel across each interval. This method is used in evaluating the performance of race cars on racing circuits such as the one at Mosport, Ontario. (See Activity 2.3.)

Choose one or more of the procedures below to obtain data for an accelerated motion.

Procedure A Ticker Tape and Timer

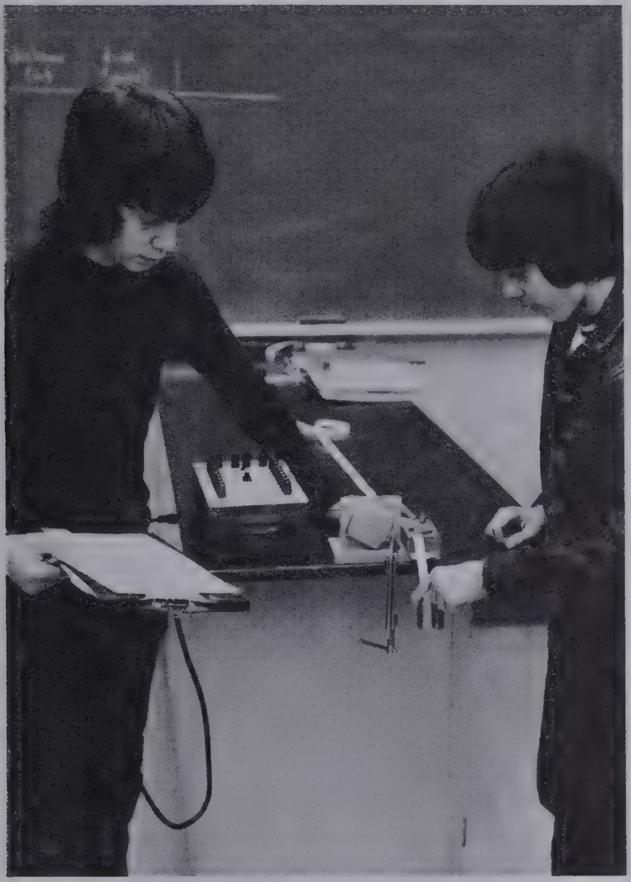


Figure 2.12

Set up the apparatus as shown in Figure 2.12.

Start the timer and let the object fall. If you want time measurements in seconds, calibrate the timer as in Experiment 2.1. Otherwise, choose your own arbitrary time unit such as *1 tock = time-interval required to make 5 dots.*

On the tape, mark off and measure the distance travelled by the mass at regular intervals and tabulate your results as shown in the previous experiment.

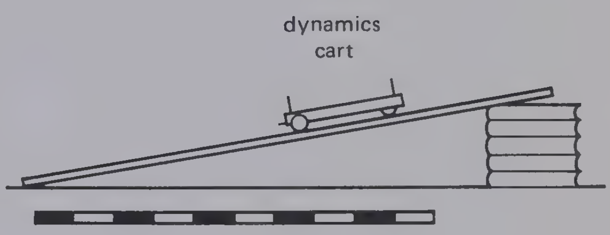


Figure 2.13

Procedure B Photograph

Take a strobe or blinky photograph for a motion such as that of a cart rolling down a straight ramp as shown in Figure 2.13.

Measure the positions of the cart from the photograph and tabulate your data as shown in Experiment 2.1.

Procedure C Stopwatches

In this part you can study the motion of a Hot Wheels car down a ramp or a slot car which is accelerated.

Mark off equal distances as far apart as possible along a straight section of the track. At least five "mileposts", each marking an interval should be used.

You will need six people, one to launch or control the car at the start, and five others, each with a stopwatch at one of the five "mile-posts". Be certain all the stopwatches are at zero. Then use a countdown ... 3, 2, 1, zero, to synchronize the starting of the car and the watches. Stop each watch as the car passes its milepost. Tabulate your data in a table similar to that used in Experiment 2.1.

Analysis

Plot a distance-time graph of the motion.

Q1 Compare characteristics of this distance-time graph and the one obtained for uniform motion in Experiment 2.1.

In order to examine the behaviour of the object's speed during the run, you will need to determine its speed at various times. Find the instantaneous speed at the end of several equal time-intervals by finding the slope of the tangent to the position-time graph at each of these points. This procedure is shown in the text on page 21. Tabulate the values.

Plot a graph of speed (on the vertical axis) versus time (on the horizontal axis).

Q2 Describe the general shape of the speed-time graph. Also relate any important deviations from the general shape of the graph to the motion which caused them.

How is the speed changing? We can find the rate of change of speed from the slope of the speed-time graph. Choose a fairly straight section on your speed-time graph and find the slope of this section.

Q3 What are the units for the slope of the speed-time graph?
Q4 What quantity does the slope of the speed-time graph represent?

Finally, find the total area under the speed-time graph.

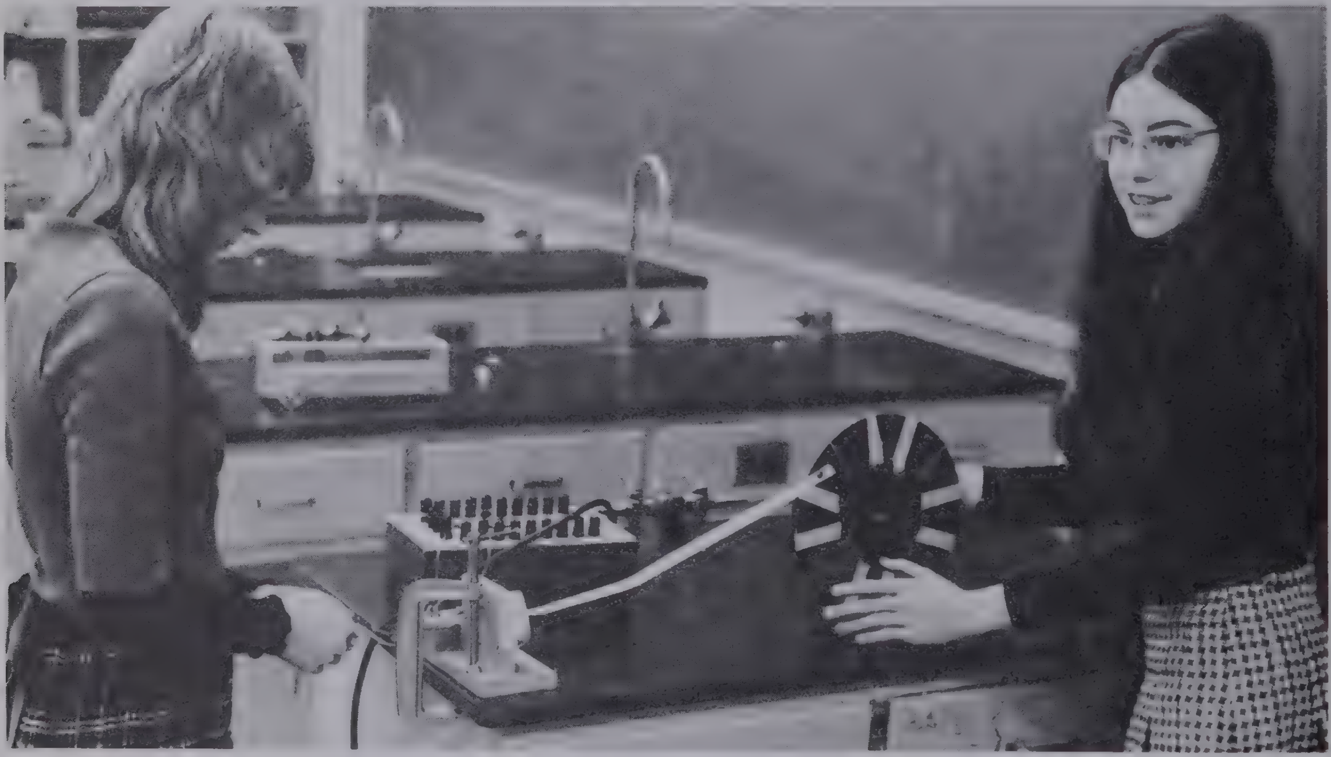
Q5 What does the numerical value of the area under the speed-time graph represent? How can you check your results?

Experiment 2.3 Motion of a Wheel

As you ride down the road at constant speed, on a bicycle, how does the speed of a point on the rim of the wheel behave? In this experiment you will investigate the motion suggested in this question.

A hand stroboscope can be used as a wheel. To analyse the motion, attach approximately 2 metres of ticker tape to a point near the rim of the strobe with a thumbtack. Make the connection loose enough so that the tape moves freely and does not wind around the strobe as it is pulled along. In this way you will have a record of the forward motion of the point but

Figure 2.14



not of its circular motion.

Run the tape through a vibrating timer. Then start the timer and roll the strobe along the table at constant speed. Repeat with another piece of tape at another speed so that you and your partner each have a tape to analyse.

If you want to know the time-interval in seconds, calibrate the timer as shown in Experiment 2.1.

On the tape, mark off and measure the distances travelled at regular time-intervals and tabulate your results.

Analysis

Plot a distance-time graph for the motion.

Q1 Using your distance-time graph, describe the motion of the point on the rim.

Calculate the speed of the point at regular intervals and tabulate the results. From the data, plot a speed-time graph for the motion.

Q2 In what ways is your speed-time graph different from one for an object moving (a) with uniform motion? (b) with constant acceleration?

Q3 Using your data, describe the behaviour of the speed and acceleration of the point on the rim during one complete revolution of the wheel. Also describe the speed and acceleration for one complete revolution of a point at the centre of the wheel. Compare the motion of these two points.

Q4 Is the point on the rim ever stopped? Discuss.

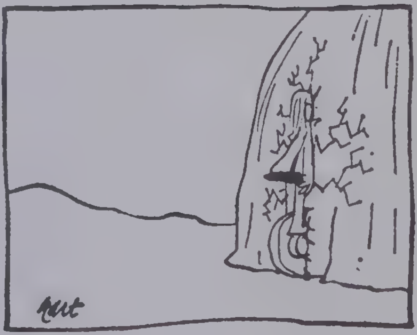
Additional Questions

Q5 What kind of results would you obtain for a point mid-way between the rim and the centre of the strobe. If you are not sure, try it.

Q6 Find the total distance travelled by the strobe from the speed-time graph. Use this distance and the total elapsed time to calculate the average speed of the point on the rim. How does v_{av} compare with the maximum speed of the point?

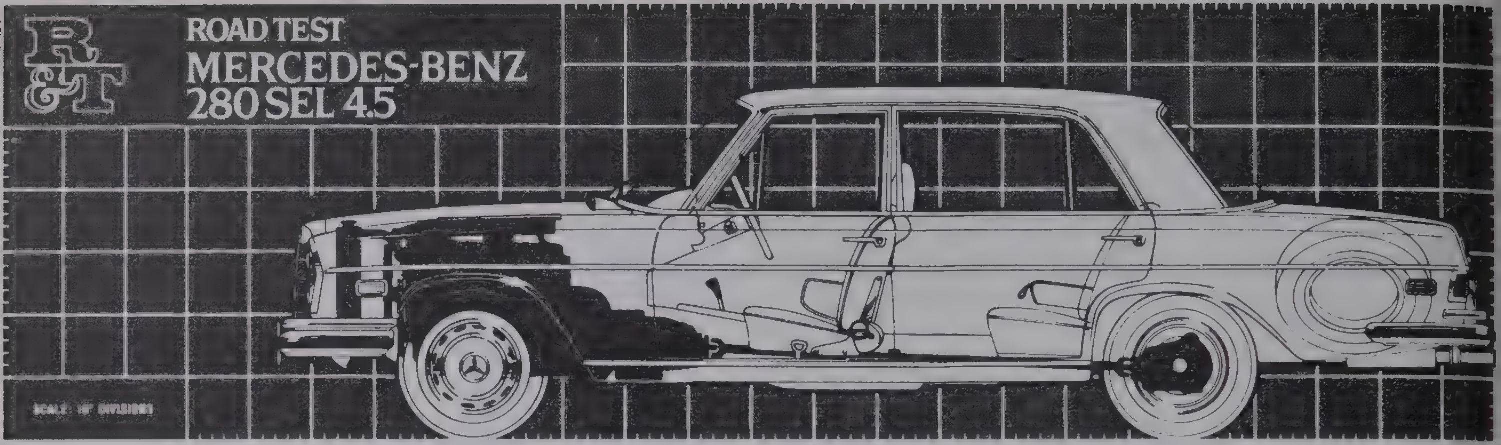
Activities

> Nothing in nature is more ancient than motion . . . yet I have discovered that there are many things of interest about it which have hitherto been unperceived.
>
> Galileo Galilei



ROAD TEST

MERCEDES-BENZ 280 SEL 4.5



CHASSIS & BODY

Layout	front engine/rear drive
Body/frame	unit steel
Brake type	10.8-in. vented disc front, 11.0-in. vented disc rear; vacuum assisted
Swept area, sq in	456
Wheels	steel disc, 14 x 6J
Tires	Phoenix 185V-14
Steering type	recirculating ball, power assisted
Overall ratio	15.7:1
Turns, lock-to-lock	3.0
Turning circle, ft	40.4
Front suspension:	unequal-length A-arms, coil springs, tube shocks, anti-roll bar
Rear suspension:	single-low-pivot swing axles, trailing arms, coil springs, tube shocks

PRICE

List price, east coast	$10,447
List price, west coast	$10,627
Price as tested, west coast.	$10,939

Price as tested includes standard equipment (automatic transmission, air conditioning, power steering & brakes, tinted glass, rear window heat), electric sunroof ($312)

CALCULATED DATA

Lb/bhp (test weight)	17.8
Mph/1000 rpm (3rd gear)	22.6
Engine revs/mi (60 mph)	2650
Piston travel, ft/mi	1480
R & T steering index	1.21
Brake swept area sq in/ton	222

ENGINE

Type	sohc V-8
Bore x stroke, mm	92.0 x 85.0
Equivalent in.	3.62 x 3.35
Displacement, cc/cu in.	4520/276
Compression ratio	8.0:1
Bhp @ rpm	230 @ 5000
Equivalent mph	113
Torque @ rpm, lb-ft	279 @ 3200
Equivalent mph	72
Fuel injection.	Bosch electronic
Type fuel required	regular, 91-oct
Emission control	fuel injection, engine mods

DRIVE TRAIN

Transmission automatic; torque converter with 3-sp planetary gearbox

Gear ratios: 3rd (1.00)	3.23:1
2nd (1.46)	4.71:1
1st (2.31)	7.47:1
1st (2.31 x 1.96)	14.62:1
Final drive ratio	3.23:1

GENERAL

Curb weight, lb	3775
Test weight	4110
Weight distribution (with driver), front/rear, %	56/44
Wheelbase, in	112.2
Track, front/rear	58.4/58.5
Overall length	196.9
Width	71.3
Height	56.7
Ground clearance	5.9
Overhang, front/rear	32.8/51.9
Usable trunk space, cu ft	17.0
Fuel tank capacity, U.S. gal	21.5

ACCOMMODATION

Seating capacity, persons	5
Seat width, front/rear	2 x 23.0/58.5
Head room, front/rear	39.0/35.5
Seat back adjustment, degrees	80

ROAD TEST RESULTS

ACCELERATION

Time to distance, sec:	
0–100 ft	4.3
0–250 ft	6.7
0–500 ft	10.0
0–750 ft	13.0
0–1000 ft	15.4
0–1320 ft (¼ mi)	18.1
Speed at end of ¼ mi, mph	78.5
Time to speed, sec:	
0–30 mph	4.7
0–40 mph	6.5
0–50 mph	8.6
0–60 mph	11.2
0–70 mph	14.8
0–80 mph	19.0
0–100 mph	32.7
Passing exposure time, sec:	
To pass car going 50 mph	6.1

FUEL CONSUMPTION

Normal driving, mpg	11.5
Cruising range, mi.	245

HANDLING

Speed on 100-ft radius, mph	31.2
Lateral acceleration, g	0.649

BRAKES

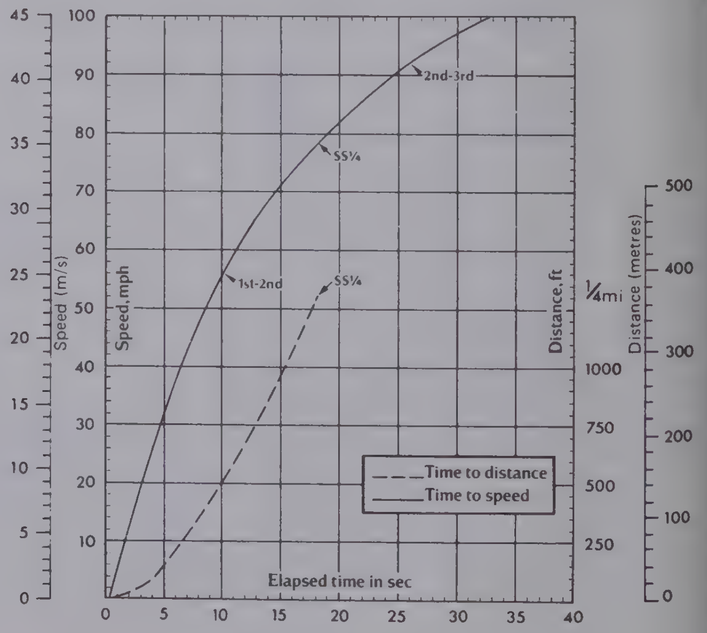
Panic stop from 80 mph:	
Max. deceleration rate, % g	87
Stopping distance, ft.	320
Control	very good
Pedal effort for 50%-g stop, lb	20
Fade test: percent increase in pedal effort to maintain 50%-g deceleration rate in 6 stops from 60 mph	nil
Parking: Hold 30% grade?	yes
Overall brake rating	very good

SPEEDS IN GEARS

3rd gear (5200 rpm)	118
2nd (5800)	90
1st (5800)	56

SPEEDOMETER ERROR

30 mph indicated is actually	27.5
40 mph	38.0
50 mph	48.0
60 mph	58.0
70 mph	67.5
80 mph	76.0
100 mph	95.0
Odometer, 10.0 mi	9.8



Activity 2.1 Road Test of a Mercedes-Benz 280 SEL 4.5

The Mercedes-Benz 280 SEL 4.5 is a luxury car which performs well. In order to evaluate a car such as this one, experts measure data such as speed and acceleration. The results of the road test done by *Road and Track* magazine, November 1971, for the Mercedes-Benz 280 SEL 4.5 are shown. You can study the data and judge the car for yourself. Some questions are provided to help guide your analysis. The speed and distance scales in metres per second and in metres have been added so that you can work in metric units.

Q1 When is the car's acceleration greatest, least, according to the speed-time graph?
Q2 How much time does it take the car to travel the quarter mile according to the distance-time graph?
Q3 What is the acceleration at 10 seconds after beginning the test run?
Q4 Plot an $a - t$ graph for the test run. (You can simplify this task by treating each 5-second segment of the speed-time graph as approximately a straight line.)
Q5 Under the heading *Brakes* some interesting data are shown for a panic stop from 36 m/s (80 mph). Sketch speed versus time and distance versus time graphs for such a panic stop. (State any assumptions.) Find the rate of deceleration from one of your graphs.
Q6 How much have cars improved since 1971? Compare results from a current road test with those for the Mercedes.

Activity 2.2 Doing Your Own Road Test

You can do your own road test to evaluate the performance of a bike or car. Some hints for the proper procedure suggested by a Canadian racing and rally driver, Hunter Floyd, are as follows:

> "Road test data for acceleration as shown in magazines such as *Road and Track* (see the road test results for a Mercedes 280 SEL 4.5) can be obtained for any vehicle by using a stopwatch. The validity of the results depends on the accuracy of distance, time, and speed measurements.
>
> Distances from the starting point can easily be determined by tape measure and then indicated by a painted line or stake. Use a level, straight, testing area. Be sure the position of the person recording the time, is the same relative to the distance indicators each time the stopwatch is punched.
>
> For acceleration tests, the indicated terminal speed is usually different from the actual speed, due to speedometer error. Note that speedometer error has no direct variability, (see Mercedes test), so that each terminal speed you wish to measure must be verified on its own. Verifying speedometers takes a little time and again involves a known distance, for example, the 400m or 0.400 km (0.25 mi), you already have measured. Drive the vehicle over the known distance at a steady *indicated speed,* for example, 50 km/h (approximately 30 mph) and record the time taken to cover it. By use of the formula,
>
> $$\text{average speed in km/h} = \frac{\text{distance in km}}{\text{time in seconds}} \times 3600,$$
>
> you can see what the *actual speed* is.
>
> Tabulate both speeds. Make further runs to determine the indicated and actual speeds you are using in your acceleration tests.
>
> Given verified distances and speeds you can do either *Time to Distance* or *Time to Speed* data charts and graphs. To further increase accuracy, run your tests in both directions and average results to correct for any slight gradient or wind factors."

Activity 2.3 Analysis of a Race

Hunter Floyd, a Canadian racing driver, has had a lot of experience preparing and racing cars in rallies and on tracks such as the one at Mosport, Ontario. Below he explains how physics is used in getting ready for a race.

> "Preparation of certain cars for competition on road circuits such as Mosport, confronts the driver with the paradox; *designing the car for too great a speed on the straightaway will actually reduce overall lap times.* This paradox may be illustrated by my experience with an Abarth Zagato 850 Twin Cam, a sports car about the size of an M.G. Midget

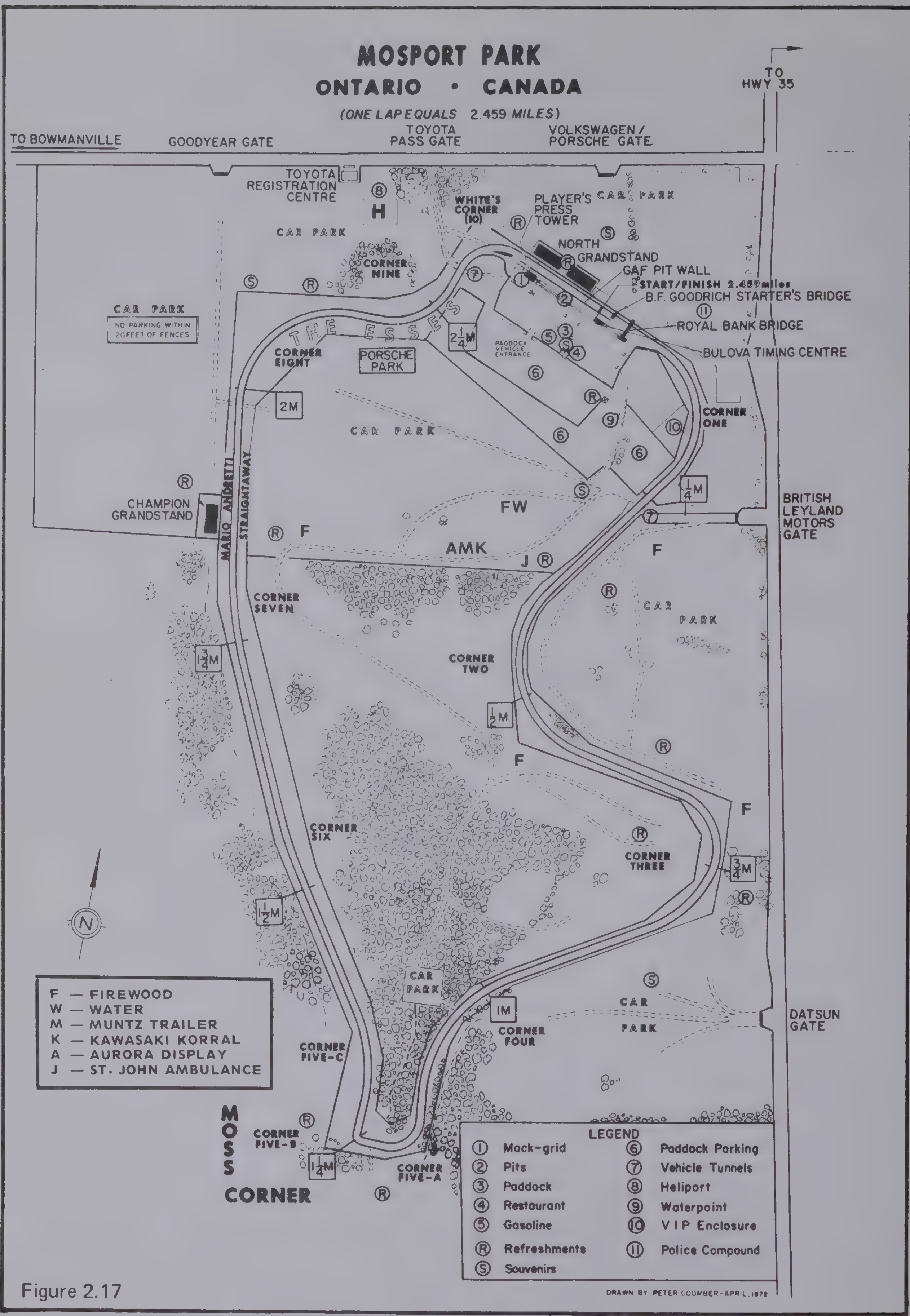

MOSPORT PARK
ONTARIO • CANADA
(ONE LAP EQUALS 2.459 MILES)
TO HWY 35
TO BOWMANVILLE
GOODYEAR GATE
TOYOTA PASS GATE
VOLKSWAGEN / PORSCHE GATE
TOYOTA REGISTRATION CENTRE
H
CAR PARK
CORNER NINE
WHITE'S CORNER (10)
PLAYER'S PRESS TOWER
NORTH GRANDSTAND
GAF PIT WALL
START/FINISH 2.459 miles
B.F. GOODRICH STARTER'S BRIDGE
ROYAL BANK BRIDGE
BULOVA TIMING CENTRE
CAR PARK
NO PARKING WITHIN 20 FEET OF FENCES
THE ESSES
CORNER EIGHT
PORSCHE PARK
2¼M
PADDOCK VEHICLE ENTRANCE
2M
CAR PARK
CORNER ONE
¼M
CHAMPION GRANDSTAND
MARIO ANDRETTI STRAIGHTAWAY
FW
AMK
J
F
BRITISH LEYLAND MOTORS GATE
CORNER SEVEN
CAR PARK
1¾M
CORNER TWO
½M
CORNER SIX
CORNER THREE
¾M
1½M
CAR PARK
1M
CORNER FOUR
CAR PARK
DATSUN GATE
CORNER FIVE-C
CORNER FIVE-B
1¼M
CORNER FIVE-A
MOSS CORNER
N
F — FIREWOOD
W — WATER
M — MUNTZ TRAILER
K — KAWASAKI KORRAL
A — AURORA DISPLAY
J — ST. JOHN AMBULANCE
LEGEND
① Mock-grid
② Pits
③ Paddock
④ Restaurant
⑤ Gasoline
Ⓡ Refreshments
Ⓢ Souvenirs
⑥ Paddock Parking
⑦ Vehicle Tunnels
⑧ Heliport
⑨ Waterpoint
⑩ V I P Enclosure
⑪ Police Compound
DRAWN BY PETER COOMBER - APRIL, 1972

Figure 2.17

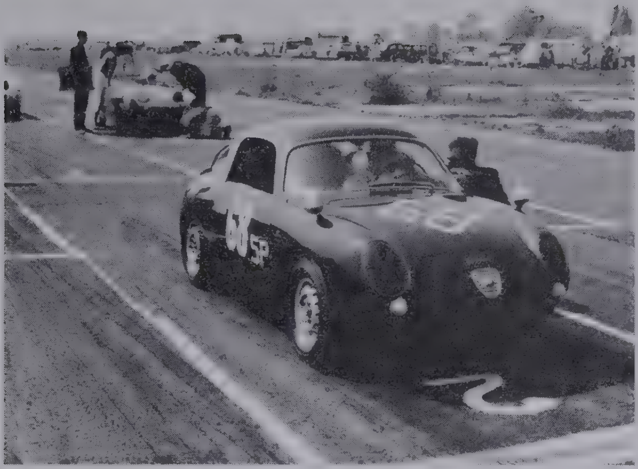


Figure 2.16

The limiting factor of performance was a maximum safe engine speed of 8000 revolutions per minute (rpm) which was easily attainable but never exceeded. I had two different axle gearsets available; with set A, 8000 rpm in top gear gave a speed of 120 mph; with set B, 8000 rpm in top gear gave only 112 mph. The acceleration with set B was greater than with set A. Timing on practice laps showed no difference in elapsed time of 92.75 seconds for the segment of the circuit shown in Figure 2.17 from the two mile marker 2M entering Corner Eight to the 1¼ M marker entering Corner Five-B, regardless of which gearset was used. Note that the fastest point on this segment was downhill through Corner Four which was negotiable at 117 mph. There was, however, a significant difference in the elapsed time on the other segment of the track from Corner Five-B's 1¼ M marker to the 2M marker of Corner Eight. Here the track runs uphill from about Corner Five-C to Corner Seven, then slightly downhill toward Corner Eight.

This topography is a factor in the following recorded data for this segment.

Marker	Speeds (mph)	
	Gearset A	Gearset B
1¼ M	40	40
1½ M	90	100
1¾ M	100	112
2 M	120	112

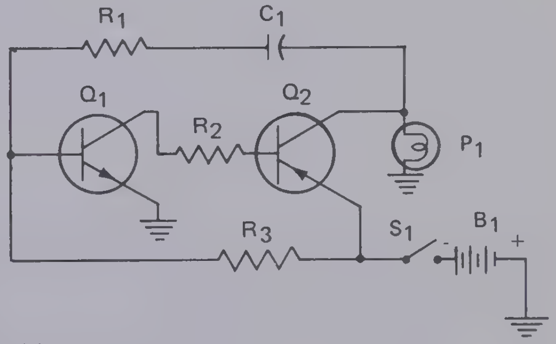
From these data and the map in Figure 2.17, you can determine which gearset gives the least elapsed time. (Assume acceleration between recorded speeds to be constant.)"

To find which gearset is better for a race consider the following data for each gearset.

(a) Average speed between markers.
(b) Elapsed time between markers.
(c) Acceleration between markers.
(d) Total elapsed time for the segment from 1¼ M to 2M.
(e) Total elapsed time for one complete lap of the circuit.
(f) Average speed for the segment from 1¼ M to 2M.
(g) Average speed for one complete lap of the circuit.

Activity 2.4 Making a DC Blinky

This is an activity for the student with some experience in putting together a simple electric circuit. In Figure 2.18 is shown the design of a small portable battery-powered light which blinks at a rate of 1 blink per second. The "blinky" can be attached to an object and photographed in a darkened room. The photograph will show the motion of the object for the time during which the camera shutter is left open.



Parts List

Transistors
Q_1 2N438A or equivalent
Q_2 2N565 or equivalent
Resistors (ohms)
R_1 10k Ω
R_2 1.2 Ω *
R_3 0.82 meg Ω

Miscellaneous
P_1 2 v 50-60 mA * (GE # 49)
S_1 SPST
B_1 6-volt DC
*important values
C_1 100 μfd (for approximate frequency of 1 hertz).

Figure 2.18

Activity 2.5 The Path of a Perseid Meteor

The study of meteors is one of the most important ways of gathering information about the earth's upper atmosphere, as well as being of immediate scientific interest. Some important quantities to be determined regarding any single meteor are the direction from which it came, the heights at which it appeared and disappeared, and its speed. The purpose of this exercise is to indicate how such data may be obtained.

Graphical methods will be substituted for the mathematical ones customarily used by professional astronomers.

DATA

A number of meteors were observed during the 1934 Perseid meteor shower simultaneously from the David Dunlap Observatory and from a temporary station near Brampton, Ontario. The Brampton station is 35.04 kilometres distant from D.D.O.; the azimuth of Brampton from D.D.O. is 232.7 degrees. (See map.)

The meteor whose path relative to the earth is to be determined, was observed at 0^h40^m EST, 13 August 1934. Following are the necessary data.

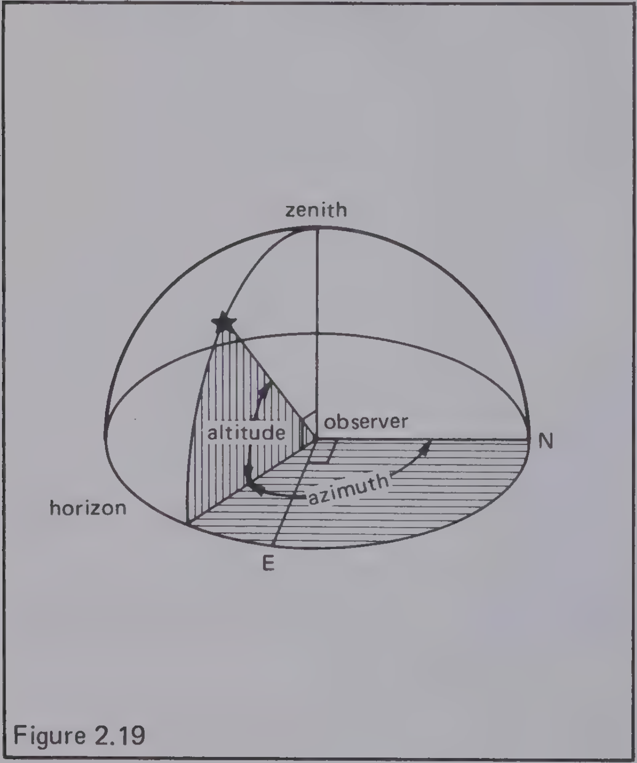


Figure 2.19

Table 2.2

	Beginning Point		Ending Point	
	Alt.	Az.	Alt.	Az.
From D.D.O.	60.3°	141.1°	46.1°	169.2°
From Brampton	54.6°	107.0°	49.4°	134.3°

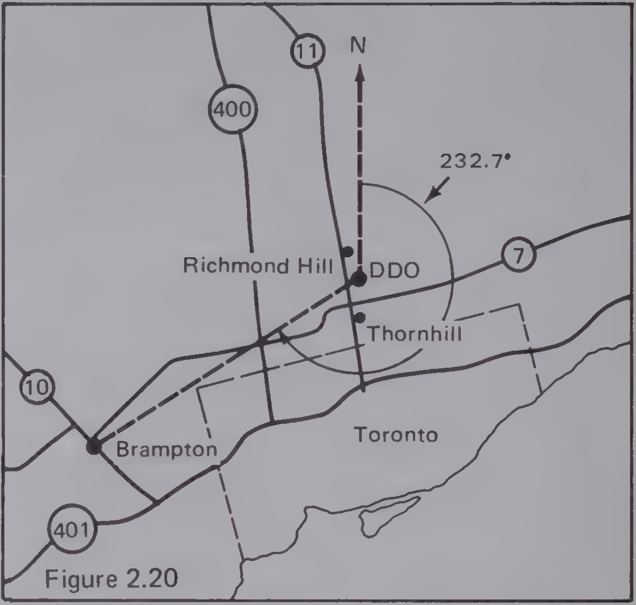


Figure 2.20

Procedure

1. Using a suitable scale, lay off on a piece of squared paper the relative positions of D.D.O. and Brampton. Be sure to indicate the scale and the direction of North.
2. Lay off and label the azimuths of the beginning of the trail from each station. The point of intersection is the point directly beneath the position where the meteor first was visible.
3. Do the same for the end-point. Label the points.
4. Along the edge of a piece of cardboard, lay off two line-segments *AB* and *BC*, making these segments respectively equal to the distances from D.D.O. to the beginning point, and from Brampton to the beginning point. Erect a perpendicular at point *B*. From *A*, lay off an angle equal to the altitude of the beginning point as seen from D.D.O., and from *C*, similarly for Brampton. When this triangle is cut out and folded along the perpendicular, it can be set up over the ground-plan to show the position of the beginning point of the meteor trail. Label all lines and points carefully.

Figure 2.21

5. Follow a similar procedure for the end-points of the meteor trail.
6. Assemble the figure and attach a bit of yarn to represent the meteor's actual path relative to the surface of the earth.

Q1 Where are the heights of the beginning and ending points of the meteor? How do these compare with average values?
Q2 What is the length of the visible path of the meteor in kilometres? If the elapsed time from beginning to ending was 0.6 seconds, about what speed in kilometres per second did the meteor have? Is it likely that any fragment of it reached the earth? Why?

Activity 2.6 Your Own Motion Experiment

Design experiments to measure the average speeds of some of the following objects in motion.
(a) A baseball thrown from outfield to home plate.
(b) The wind.
(c) A cloud.
(d) A raindrop.
(e) A kicked or thrown football.
(f) The tip of a swinging baseball bat.
(g) An insect walking or a worm crawling.
(h) A camera shutter opening and closing.
(i) An eye blinking.
(j) A whisker growing.
(k) The vibrating speed of the centre of a vibrating guitar string.
(l) A bird flying.

Try at least one of these experiments and report your procedure and results.

Activity 2.7 Monitoring a Car's Performance

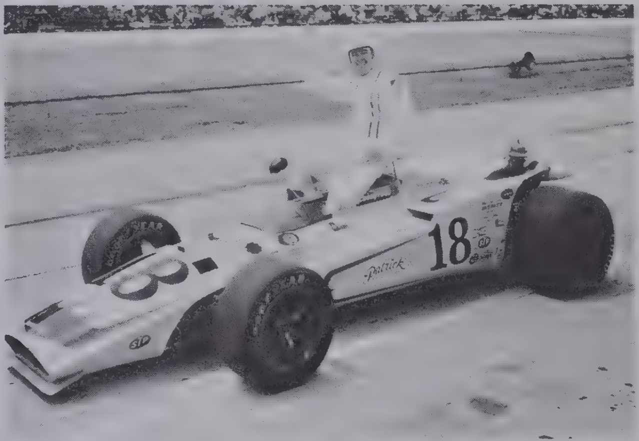
Performance data for the fastest cars are recorded with the aid of multi-channel telemetry equipment. A light weight (2 kg) sending unit is mounted on the car and its signal is received in the pit area. The crew is able to monitor several characteristics of the car's behaviour simultaneously.

Six telemetry traces are shown for a test run of a Michener-Patrick Special, powered by a turbo-charged Offenhauser engine, and driven by Johnny Rutherford on the track at the Ontario Motor Speedway in Ontario, California. This is no ordinary car, having top speeds greater than 320 km/hr (200 mph). In our sample, traces *A, B,* and *C* show the up and down motion of individual wheels and are used to determine optimum spring rates and shock absorber settings. *D* and *E* record the car's acceleration and its instantaneous speed. On all of these graphs, calibration is shown for a single vertical scale unit. The *T* tracing shows time in seconds.

Ontario Motor Speedway is similar to Indianapolis in its layout. There are two long and two short straightaways joined by four corners *C1, C2, C3,* and *C4*.

Traces *D* and *E* can be explained in terms of the car's position and performance on the track. Each segment begins just as the car reaches maximum speed on the front straightaway. What is this maximum? The car slows for the first corner *C1*, accelerates through it and down the short straight, decelerates going into *C2*, accelerates through it and so on through the remaining corners and straights to complete one lap.

Figure 2.22



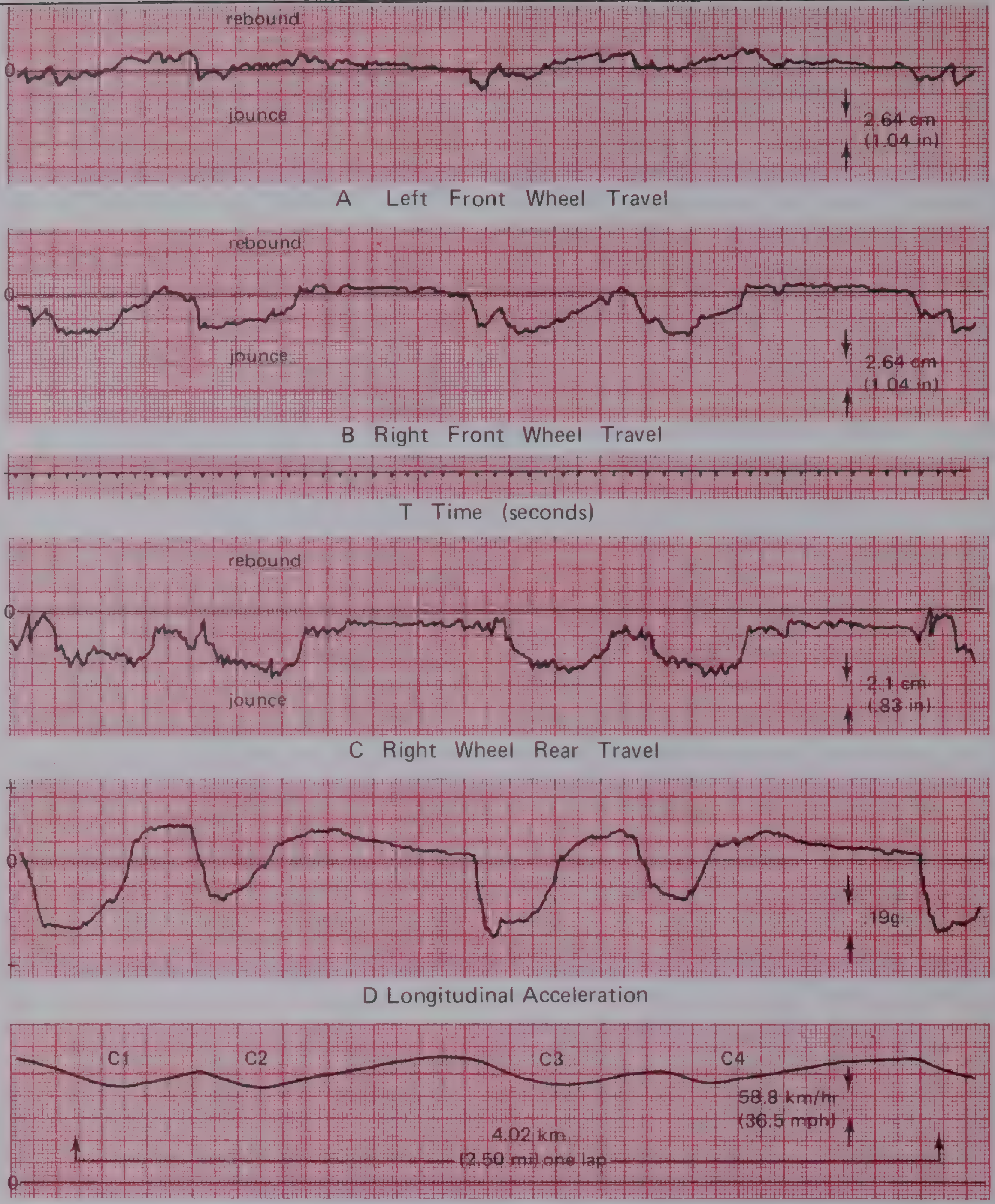


Figure 2.23 E Speed

Using the tracings you may determine the following:

(a) acceleration at any point. (Trace *D* is calibrated in *g*'s; one *g* is equal to the acceleration due to gravity.)

(b) instantaneous speed at any point.

(c) maximum and minimum speeds during the lap.

(d) average speed for any segment of the lap.

(e) lap time

(f) lap speed

(g) slowest and fastest corner.

You can also use the data to calculate times taken in covering various segments of a lap and to plot a distance-time graph for one lap.

Chapter 3. Galileo Describes Motion

> "Science came down from heaven to earth on the inclined plane of Galileo."
>
> Author Unknown

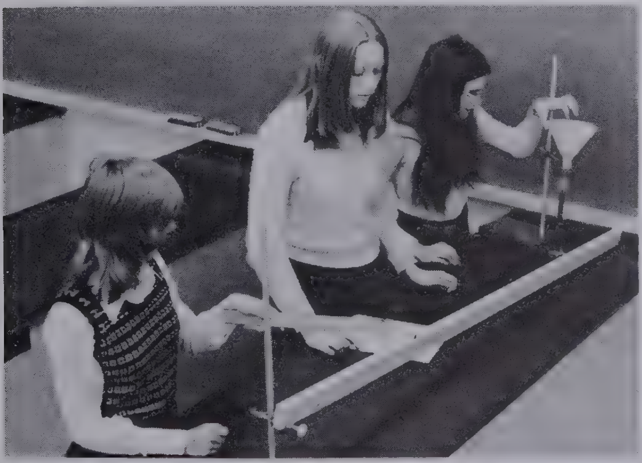
The first experiment in this chapter was originally described by Galileo in his book, *Two New Sciences.* He was seeking to test his hypotheses about accelerated motion. Your purpose and procedure will be similar to those of Galileo. You will measure the motion of a ball rolling down an inclined plane. To capture the flavour of the original experiment, you will measure time, not with a stopwatch, but with a water clock, which was the most accurate type of timepiece available to Galileo. With the same apparatus, you can obtain data with which to calculate the acceleration of a falling object.

Before beginning you should understand the reasoning which led Galileo to do the experiment. His thinking is developed in Chapter 3, Sections 3.7 through 3.9 of the text. It may help if you summarize the main points before starting.

Experiment 3.1 Galileo's Inclined Plane

The inclined plane apparatus is shown in Figure 3.1. Keep the angle between the table and inclined plane less than 10 degrees.

Figure 3.1



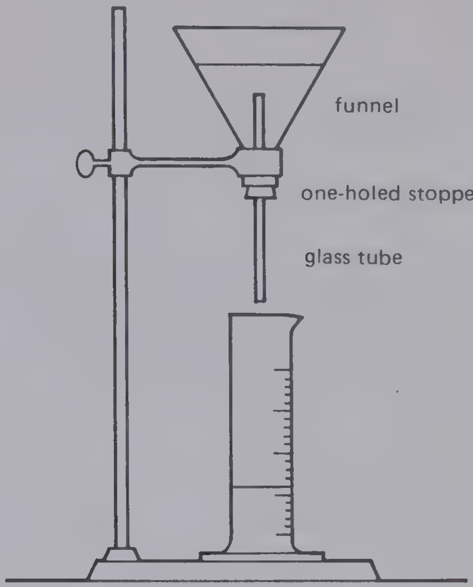


Figure 3.2

Devise a water clock like the one described by Galileo in the statement below taken from the text.

"For the measurement of time we employed a large vessel of water placed in an elevated position; to the bottom was soldered a pipe of small diameter giving a thin jet of water, which we collected in a cup during the time of each descent . . . the water thus collected was weighed."

One possible arrangement is shown in Figure 3.2.

Procedure

Prepare a table with headings similar to those shown in Table 3.1, for recording the times taken by the ball when rolling various distances down the incline.

Fill the water clock and keep it from running by stopping the top of the glass tube with your finger.

Since the amount of water which flows from the clock is directly proportional to the time-interval, and since 1 ml of water weighs 1 gram, you can calculate the time from the volume of water in the graduated cylinder as suggested by Galileo. Your time units will be ml of water.

Table 3.1

Distance (m)	Times for repeated trials (ml of water)	Average time T (ml of water)	T^2 (ml^2 of water)	d/T^2 ( )
0.10				
0.20				
0.30				

Hold the ball in the starting position with a ruler and launch it by pulling the ruler aside. This will avoid a push or pull on the ball so that its initial speed will be zero. Start and stop the water-flow of the clock at the beginning and ending of the distance-interval.

Measure the water volume in the cylinder and be sure to return the water to the clock so as to maintain the same level throughout the experiment. Repeat the run three more times and average the results to obtain a reliable value of the time-interval.

Keeping the ramp at the same angle, measure the time of descent for several distances and tabulate your data. This will give you sufficient data to test Galileo's hypothesis.

CALIBRATING THE WATER CLOCK

Calibrate your water clock in seconds with a stop-watch by measuring the time in seconds required for 50 ml of water to flow from the clock.

Analysis

Q1 What relationship between d and T did Galileo predict for this experiment?

Use your data to determine if Galileo's hypothesis is confirmed by your experiment. One way to do this is to calculate values of d/T^2 for each trial and compare the values.

An alternative method for testing the hypothesis is to plot a graph of distance versus time squared. Use your graph to reach a conclusion.

Q2 How much time in seconds is represented by 1 ml of water?

Q3 Galileo claimed that his water clock was accurate to within one-tenth of a pulse beat. Is your water clock that accurate? Discuss, using your measurements.

Q4 How does the accuracy of Galileo's water clock compare with that of a modern stopwatch?

Q5 If you used a ball of different mass, would the results be different? Why?

Additional Activities

1. *Acceleration on the Ramp*

Hopefully, in the experiment you have found that d/T^2 is constant. There is a close relationship between the value of the constant and the acceleration of the ball. From the equations of motion (Chapter 2 Section 2.2 in the text), we see that,

$$d = v_i T + \frac{1}{2} a T^2 .$$

On the incline, the initial speed is zero so that,

$$d = \tfrac{1}{2} a T^2 .$$

Solving for a,

$$a = \frac{2d}{T^2} .$$

Use this relationship to calculate the acceleration of the ball on the ramp by finding an average of the d/T^2 values for the various runs, or the slope of your d versus T^2 graph.

2. *Extending, to find Acceleration due to Gravity*

This extension is difficult but worthwhile if you have the time.

As the slope of the ramp increases, the values of d/T^2 and the acceleration also increase. But the ramp experiment works only for small angles of inclination (less than 10 degrees) because as acceleration increases, the time-intervals become too short for accurate measurement. Also, the ball begins to slip on the ramp. Consequently Galileo was not able to measure the acceleration due to gravity directly (when the ramp would have had to be inclined at 90 degrees). But we can measure acceleration for various small angles and extrapolate to find an expected value for 90 degrees (at which angle the object would be falling vertically).

Repeat the experiment for various angles of inclination of the ramp using the same distance along the incline each time. Record data in a table similar to Table 3.2. Use a stopwatch, or calibrate your water clock, so that you measure the time-interval in seconds.

Plot a graph of acceleration (on the vertical axis) versus the angle of inclination (on the horizontal). Leave room to extrapolate the graph to an angle of inclination of 90 degrees.

Q6 What shape is the graph?

Extend it, assuming similar behaviour at all angles from 0 to 90 degrees.

Q7 What acceleration do you predict at 90 degrees?

The accepted value of acceleration due to gravity for an object falling vertically is 9.8 m/s². Calculate the percentage errors in your result using the formula,

$$\text{percentage error} = \frac{\text{accepted value} - \text{obtained value}}{\text{accepted value}} \times 100\%.$$

Thus, if your result is 9.0 m/s², then the percentage error

$$= \frac{9.8\ \text{m/s}^2 - 9.0\ \text{m/s}^2}{9.8\ \text{m/s}^2} \times 100\%$$

$$= 8\%.$$

Q8 What are the sources of error?

Experiment 3.2 A Modern Version of Galileo's Inclined Plane

Using an air track as the incline, and timing with a stopwatch or electronic timer, you can test more rigorously Galileo's hypothesis that d/T^2 is a constant if the acceleration of an object is uniform.

Use the same procedure as outlined in Experiment 3.1. Your analysis should also follow in the same way.

Q1 Is d/T^2 constant for the air track glider?

Q2 Does an increase in accuracy change your conclusion? Do more accurate measurements reveal a flaw in Galileo's hypothesis?

Additional Activity

Extending, to find Acceleration due to Gravity

If you have time, use the procedure outlined in the previous experiment to find an acceleration value for the glider if the air track is vertical.

The accepted value of acceleration due to gravity is 9.8 m/s². Find your percentage error.

Q3 How much better were your results for this version, than for the version based on the one used by Galileo? Why?

Table 3.2

Angle of inclination (degrees)	Distance (m)	T (s)	Average T (s)	Acceleration (m/s²)
2				
4				
6				

Experiment 3.3 Measuring Acceleration due to Gravity

There are several methods of measuring acceleration due to gravity in this experiment. You will not have time to try all of them, but choose at least one.

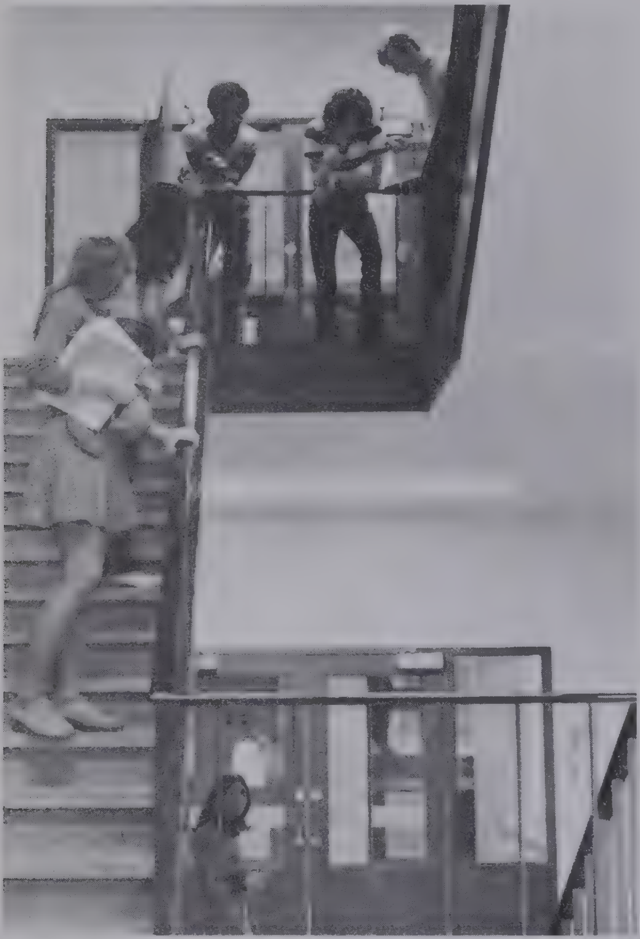


Figure 3.3

Procedure A A Long Drop

Find a stairwell or a platform from which you can drop a metal sphere at least 5 metres. Check out your drop site with your teacher before proceeding.

From equations of motion we know that $a = 2d/T^2$ for an object accelerating uniformly from rest. Consequently, you can determine the acceleration of the sphere by measuring the distance and time to fall.

Arrange to drop the ball so that your partner with the stopwatch can begin timing at the instant you drop the ball and stop timing when the ball hits the floor. If you are outside use a board so that the sphere will make an audible, sharp sound when it hits. Repeat several times and average your time results.

Calculate a value of acceleration due to gravity.

ERRORS

The accepted value of acceleration due to gravity is 9.8 m/s^2. To evaluate how well you did the experiment, find your percentage error by using the following formula.

$$\text{percentage error} = \frac{\text{accepted value} - \text{obtained value}}{\text{accepted value}} \times 100\%$$

A reasonable result agrees within 5% of the accepted value.

Q1 What are the sources of error in the experiment? Which source is most significant? Which are negligible?

LEANING TOWER OF PISA

According to legend, Galileo dropped cannonballs of various masses, from the Leaning Tower, to refute the Aristotelian theory that a heavier object falls faster than a lighter one. You could do the same sort of test by dropping a heavy and a light steel sphere simultaneously and determining which strikes the floor first, or, by dropping them separately and measuring the time for each to fall through the same distance.

Figure 3.4

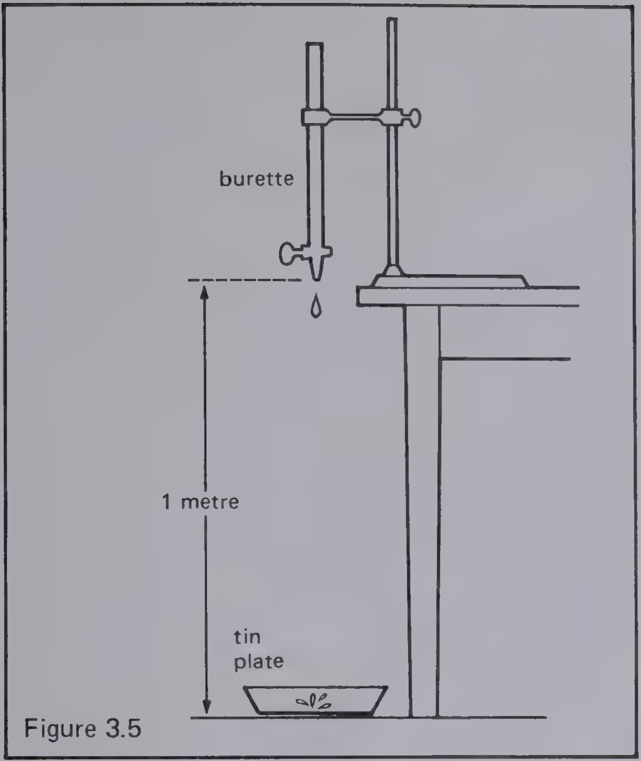


Figure 3.5

Procedure B Falling Water Drops

Set up a burette and tin plate as shown in Figure 3.5.

Fill the burette with water. Open the tap slowly to get a steady flow of *drops* and adjust the rate until one drop hits the pan at the same instant as the next drop leaves the burette. *Watch* the drop leaving the burette and *listen* to the other hit below.

Leave the tap at this position, and using a stopwatch, find the time-interval between drops, (the time for the drop to fall one metre). This can be done accurately by finding the time-interval for 20 drops and dividing to find the time-interval for one.

From equations of motion, we know that $a = 2d/T^2$ for an object accelerating uniformly from rest. Assuming that the water drop is in free fall, calculate from your observations the value of acceleration due to gravity.

Find the percentage error in your result following the steps outlined in Procedure A. Comment on possible sources of error.

If you do not have time in class to do this experiment with a burette, you can do it at home in the shower.

Procedure C Strobe Photograph of a Falling Sphere

By taking a photograph of a steel sphere falling from rest and illuminated by a stroboscope, you can obtain a record of the position of the object at equal time-intervals. You can analyse this photograph in order to obtain the sphere's acceleration due to gravity.

To get the photograph, set up a dark background such as a black cloth, in a room which can be darkened. Place a metre stick vertically at one side so that it will be in the photograph giving a convenient reference length. Then you can scale your measurements from the photograph to actual size. Set up the camera as outlined in Appendix A so that the entire drop will be in the picture. Adjust the strobe frequency to a convenient frequency which will give at least 5 images in the picture during the drop. To obtain the ball's initial position, open the shutter just before the ball is dropped. Close the shutter just after the ball hits the floor.

From the photograph measure the distance fallen by the ball to each point. Be sure to determine actual size of these distances in metres.

Find the time-interval between flashes of the strobe.

Tabulate the distance-time data. Plot a speed-time graph of your results.

Q1 What shape speed-time graph should you get for a freely falling object? Does your graph satisfy your prediction?

Measure the sphere's acceleration from your speed-time graph.

Using the method outlined in Procedure A, find the percentage error in your results.

Q2 What are sources of error in your experiment? Which source is the most significant? Which are negligible?

Procedure D Simple Pendulum

A simple pendulum can be constructed from a steel ball suspended from a support by a thread as shown in Figure 3.6.

The time T for one round-trip swing of the pendulum, depends on the length, l, of the pendulum (measured from the support to the centre of mass of the bob), and the acceleration due to gravity a_g.

Q1 Using your intuition, how do you think the time for one complete swing of the pendulum would be affected by

(a) an increase in the length of the pendulum?

(b) an increase in the acceleration due to gravity?

Check your conjecture by considering the equation below.

Although you may not have sufficient mathematical knowledge to derive the relation among t, l, and a_g, you can make use of the relation derived by others,

$$T = 2\pi\sqrt{\frac{l}{a_g}}\,.$$

If you measure T and l, you can solve for a_g by substitution.

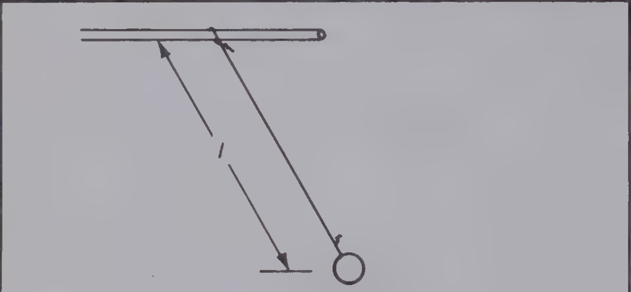


Figure 3.6

Set up the pendulum as shown. Measure and record the length l in metres.

Start the pendulum swinging at a *small* angle to the vertical. Measure the time for at least 20 complete swings (across and back) and calculate the time T in seconds for one swing. Repeat as a check.

Q2 Why is it wise to measure the time for 20 or more swings instead of a small number of swings?

Calculate a value for a_g.

Using the method outlined in Procedure A, determine the percentage error in your results.

Q3 What are the sources of error? Which source is most significant? Which are negligible?

One precaution indicated in the procedure was to use only small swings. If you have time, redo the experiment using large swings to find out what effect they have on your results.

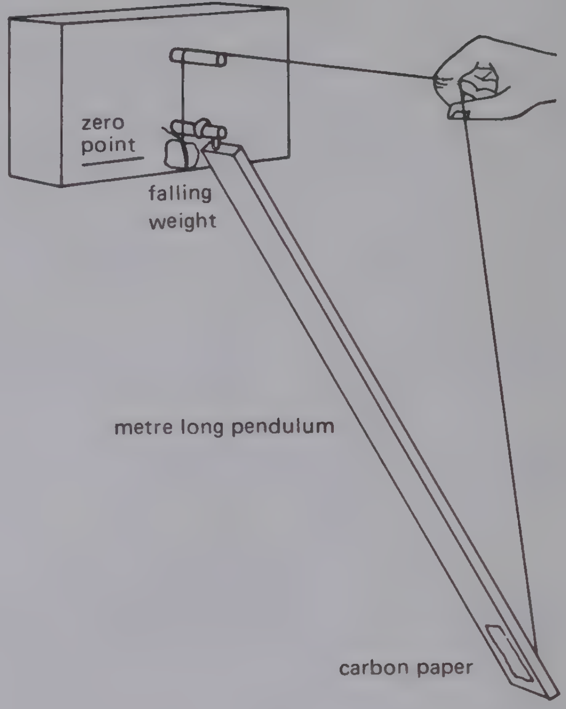


Figure 3.7

Procedure E Acceleration Apparatus

Your school may have an apparatus like the one shown in Figure 3.7, designed specifically to measure acceleration due to gravity. If it does not, you can build one using a metre stick, a screw eye, two finishing nails, a piece of smooth string approximately 2 to 3 metres long, a wood block, and a lead sinker.

Align the ball with the zero point. It should now be in a position so that if it were released, it would just graze the vertical pendulum as it falls. Now pull the stick back so that the pendulum is pulled aside about 10 cm. Burn the string so that both the weight and pendulum begin to fall simultaneously. The weight should strike the carbon paper when the pendulum reaches the vertical position.

From equations of motion for an object falling from rest and accelerating uniformly,

$$d = \tfrac{1}{2}\, a_g T^2.$$

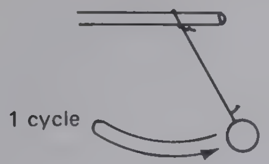


Figure 3.8

To find *d*, measure the distance from the zero point to the mark where the weight hit the carbon paper. To find *T*, time the pendulum for several complete cycles. One cycle is one complete swing from the point where it is released to the opposite side and back to the release point.

T is the time from when the string was burned and the pendulum began to fall to the instant when the falling weight collided with the pendulum making the mark. It is also the time for one-quarter of a cycle of the pendulum. From your observations, calculate a value for *T*. Finally, use the equation above to calculate a_g.

Find the percentage error in your results.

Q1 What are the sources of error in this experiment? Which source is most significant?

Procedure F Acceleration due to Gravity, Film Loop

Acceleration due to gravity can be measured from the film loop entitled *Acceleration Due to Gravity 1*. The method of analysis is outlined in *Film Loop Notes*, page 128.

Procedure G Galileo's Inclined Plane

You can also find acceleration due to gravity from an inclined plane experiment.

Use the method outlined in Experiment 3.1, Galileo's Inclined Plane Experiment. (Experiment 3.2 is similar but is done using more modern apparatus.)

Activities

> "When . . . I observe a stone initially at rest falling from an elevated position and continually acquiring new increments of speed, why should I not believe that such increases occur in a manner which is exceedingly simple and rather obvious to everybody."
>
> Galileo Galilei

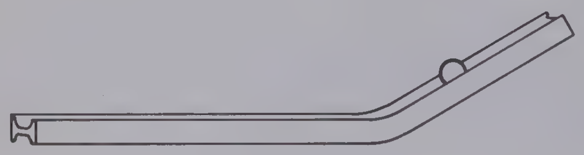


Figure 3.9

Activity 3.1 A Motion Puzzle

A combination inclined and flat plane can be formed by bending a piece of drapery track as shown in Figure 3.9 at its mid point.

Start a ball one-quarter of the way along the incline and measure the time it takes to roll down the incline; repeat and measure the time it takes to roll across the level track.

Then, start the ball at the top of the incline and measure the corresponding times. The sum of the times for each case should be the same! For example, if in the first case it takes 1 second on the incline and 2 seconds on the level, then when you start the ball at the top, it will take 2 seconds on the incline and 1 second on the level.

Explain why this happens basing your reasoning on the equations of motion.

Try to find other combinations which give similar results.

Activity 3.2 Measuring Human Reaction Time

Have your partner hold a metre stick vertically. Hold your thumb and index finger on either side of the metre stick at the bottom of the stick. As soon as the other person releases the stick, without giving you prior warning, react and catch the stick as quickly as you can. Since the stick was falling from rest, you know that $d = \frac{1}{2} a_g T^2$. Using $a_g = 9.8 \text{ m/s}^2$ and d, the distance through which the metre stick fell in the experiment, calculate T, which is a measurement of your reaction time.

Your reaction time is very important when you encounter a dangerous situation driving a car. Estimate how far you would travel at 100 km/h (approximately 60 mph) before you could react to hit the brakes.

Activity 3.3 Dime and Feather Tube

Air resistance makes it impossible to show that two objects such as a feather and a coin fall with the same acceleration. In a tube constructed as shown in Figure 3.10, this resistance can be lessened by removing most of the air and comparing rates of acceleration.

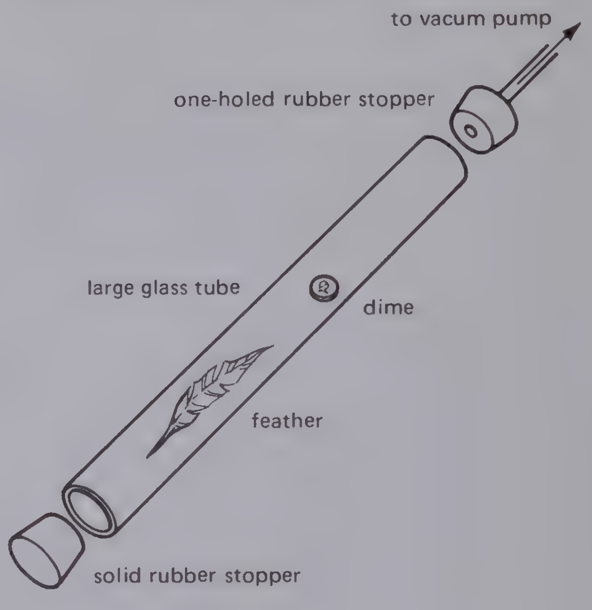


Figure 3.10

A good vacuum seal can be formed by smearing high vacuum grease on the stoppers and inserting one in each end of the tube. Pump out most of the air. Then hold the tube vertically, flip it over quickly so that the feather and coin begin to fall simultaneously. Let some of the air back in and repeat.

Activity 3.4 Egg Drop Contest

This activity, related to falling objects, can be challenging and also a lot of fun. The aim is to have a competition with your fellow students to see who can devise the most protective device in which a fresh egg can be placed, and then dropped, without breaking, straight down at least three stories (approximately 10m). There is a catch though, the contest-winning design will be the one which is the lightest in weight excluding the weight of the egg. Parachutes are not allowed, and the package cannot be larger than 10 cm x 10 cm x 10 cm.

Activity 3.5 The Importance of Air Resistance

Try to obtain a ping-pong or styrofoam ball and a steel ball of about the same diameter. Do you think they will both accelerate at the same rate? Try dropping them from a height of at least 5 metres and see if their accelerations are the same.

Objects falling in a resistant medium eventually reach a terminal velocity. Take a strobe photo of the steel ball and the ping-pong ball during the last 2 metres of a long drop and compare their motions. Find out if either reached terminal velocity.

Activity 3.6 Falling Weight

In his book, *Dialogues Concerning Two New Sciences*, Galileo, interested in falling objects, states,

> "The spaces passed over in natural motion are in proportion to the squares of the times, and consequently the spaces passed over in equal times are as the odd numbers beginning from one . . ."

That is, the distance-intervals are in ratios 1:3:5 . . .

You can test his statement by suspending weights on a string at distance-intervals which are in the ratios 1:3:5 . . .

Attach washers to a suspended string as shown in Figure 3.11.

Place a metal pie plate under the string. Then cut or burn the string at the point of suspension. Use a tape recorder to help you measure the times taken to fall. To improve your accuracy, record at a high speed and play back at a slower rate.

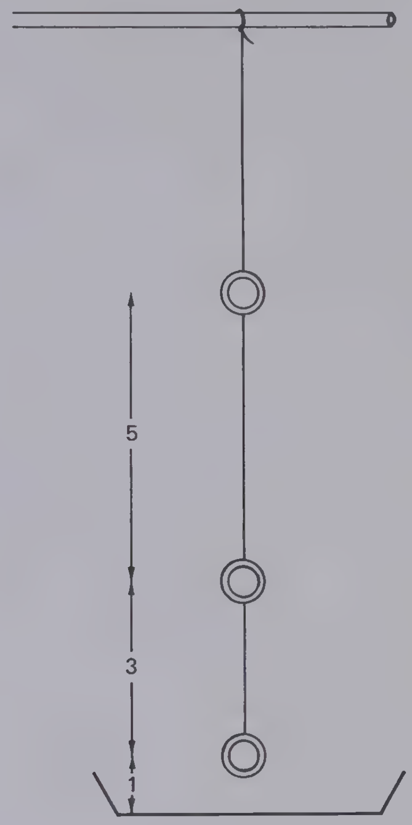


Figure 3.11

The washers should hit the pan at equal time-intervals. Why?

Activity 3.7 Acceleration of Water Drops

An interesting demonstration of acceleration due to gravity can be done using a dripping laboratory faucet and an electronic stroboscope. Set the faucet dripping at a constant rate and adjust the flash rate of the strobe so that the drops appear frozen or stopped. Measure the spacing between the stopped droplets and show that the results are consistent with Galileo's statement in Activity 3.6, and also, with the equations of motion in Chapter 2 of the text.

From these measurements and the frequency of the stroboscope, you can also calculate the acceleration due to gravity.

Film Loop Notes

Film Loop 3.1 Acceleration due to Gravity 1

This loop is a slow motion film of a bowling ball falling freely. Shown is a frame from the loop.

On the background are four dark lines marking two intervals which we will call *AB* and *CD*. The distances *AB* and *CD* are each 0.50 metres. The ball falls 1 metre before reaching *A*. The film was taken at 3900 frames per second but it will be projected at approximately 18 frames per second by the film loop projector. The slow motion factor is approximately 3900/18≃217.

With this slow motion record, it is possible to measure times and speeds far more precisely than in the actual case. Consequently, you can measure acceleration, as Galileo was unable to do, using the definition,

$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \; .$$

You can find two instantaneous velocities v_1 and v_2 at the mid-times of *AB* and *CD*, t_1 *and* t_2 respectively. Remember, if the acceleration of an object is uniform, the instantaneous velocity at the mid-time of an interval is the same as the average velocity over the intervals. (See Section 3.10 in the text for more details.)

Since the distances *AB* and *CD* are given, you need only the time measurements from the loop in order to calculate the ball's acceleration due to gravity. Time measurements from the loop will be in apparent-seconds because of the slow motion factor. For best results, calibrate the projector you are using in frames per second by timing with an ordinary watch with a second hand, the entire loop, which contains 3273 frames from start to finish. Calculate the slow motion factor.

Rerun the loop from the beginning and note the times at which the same part of the ball crosses each of the four lines *A, B, C,* and *D.* From these calculate, (a) the time between the mid-times of *AB* and *CD*, and (b) the time-intervals for the ball to travel through the ½-metre distance-intervals, *AB* and *CD*.

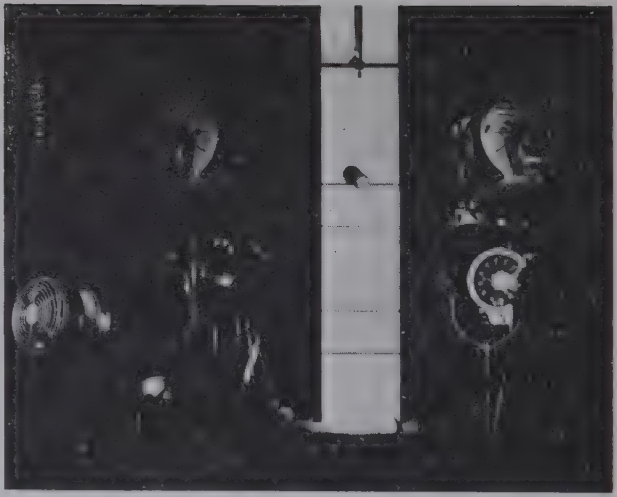


Figure 3.12

Repeat these time measurements and calculations and average your results for (a) and (b). Then convert these results to real-seconds using the slow motion factor.

Calculate v_1 and v_2. Calculate the acceleration.

The accepted value for acceleration due to gravity of falling bowling balls in Montréal, where this loop was made, is 9.8 m/s^2. Find the percentage error of your results.

Chapter 4. Kinematics: The Importance of Direction

If you did the experiments on the analysis of motion in Chapter 2 (2.1, 2.2, 2.3), you will know how to analyse the motion of an object moving in one direction. In this first experiment you will study the motion of an object whose direction of motion changes.

Experiment 4.1 Motion and Direction

Figure 4.1 is a diagram of a stroboscopic photograph of the glider on an air track set at an angle of 10 degrees above the horizontal. The glider is launched from point *A*, travels up the track, reaches a peak height at *B,* and then moves back down to the launcher. A long shutter is used on the track in front of the glider. When the glider is moving up the track, only the lower portion of the marker can be seen. At the instant the glider reaches *B,* the shutter is lowered so that on the downward trip only the top of the marker can be seen. A metre stick is placed in the picture so that you can determine actual distances. The frequency of the stroboscope is 10 hertz (cycles/S).

Procedure

Measure and tabulate the time elapsed and the corresponding positions of the glider at equal time-intervals. Measure its position relative to point *A*.

Plot a position-time graph for the motion.

Q1 What is the shape of the position-time graph?

Find the velocity of the glider at equal time-intervals and tabulate these values.

Plot a velocity-time graph.

Q2 Based on the velocity-time graph, what can you say about the acceleration of the glider for the entire motion?

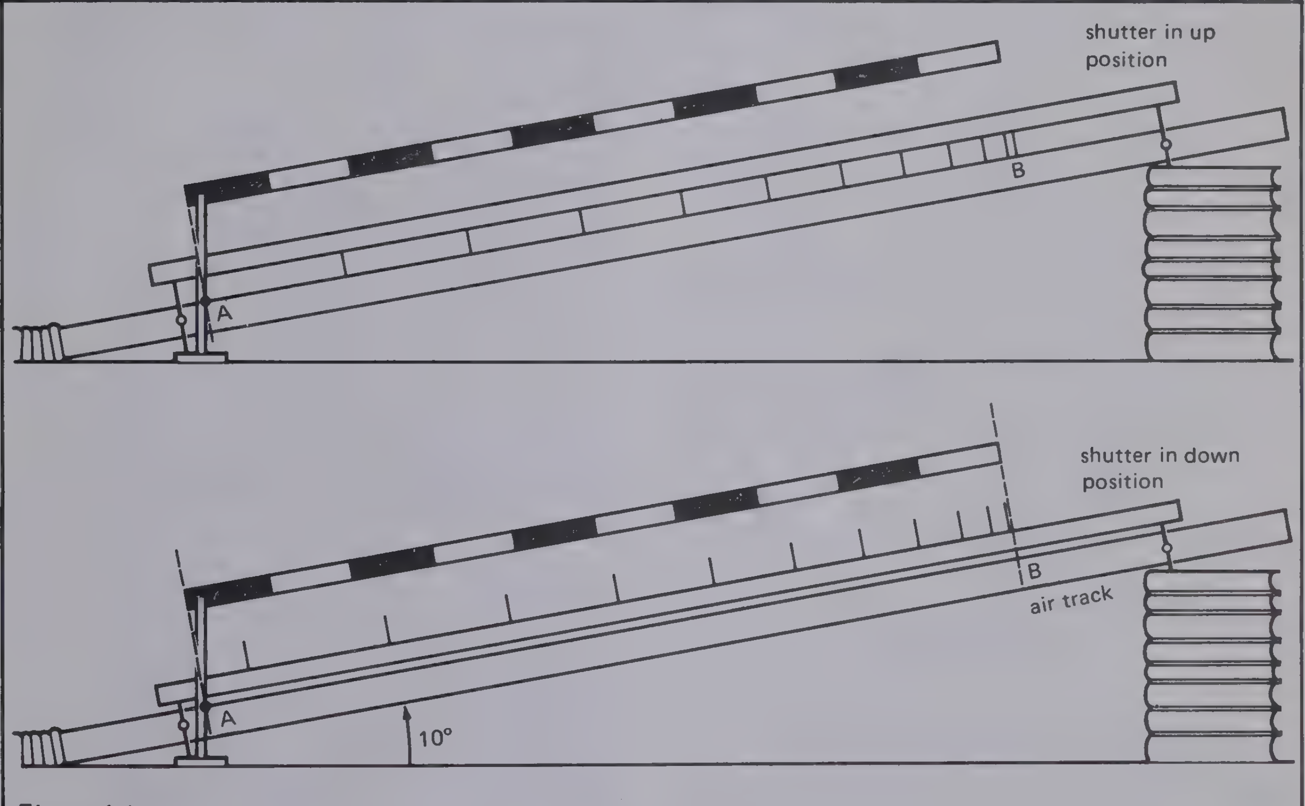


Figure 4.1

Q3 Assuming that the direction up the track is positive, complete a table like the one shown indicating by the signs (+ or −) with each of the distances, the motion up or down the track.

	Position	Velocity	Acceleration	Time
A to *B*				
B to *A*				

Additional Questions

Q4 If the measurements of position had been made relative to point *B* at the top of the track, what differences would there be in the results? Sketch *x-t*, *v-t*, and *a-t* graphs to illustrate your answer. Compare these sketches with your graphs plotted earlier.
Q5 Use the results from this experiment to measure the acceleration due to gravity.

Film Loop Notes

Film Loop 4.1 A Matter of Relative Motion

This film loop shows three events in which carts of equal mass collide. The camera takes pictures of the events from a second ramp parallel to the one on which the collision occurs. When you watch the loop, you are observing the events from the camera's frame of reference.

Watch the loop carefully. Describe each event as it happens. Try to explain what happened in each case.

The events were repetitions of the same interaction and yet they appeared to be different and perhaps to have different causes.

Q1 Why did each event appear to be different?
Q2 How can you tell which one of the carts is in motion initially?
Q3 Is the "fixed" frame of reference marked by the experimenter's tie really fixed, and if so, fixed relative to what?

As shown in the loop, the description of motion and interaction depends fundamentally on the frame of reference chosen. Go back over the film and review the events, bearing in mind the frame of reference in each case. Think about how this affects the appearance of the events.

Film Loop 4.2 Galilean Relativity — Ball Dropped from Mast of Ship

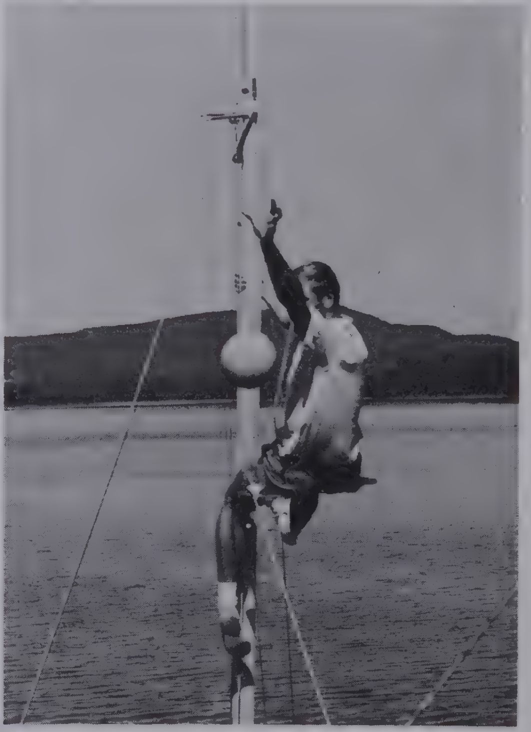
In the loop a ball is dropped three times.

Event 1. The ball drops from the top of the mast of the boat which is moving at a constant velocity.
Event 2. The ball drops from a stationary tower as the boat goes by at a constant velocity.
Event 3. A student releases the ball from the mast top as the boat moves with constant velocity.

Describe the motion of the ball as seen from two frames of reference, the boat and the dock, in each event.

Q1 In Event 1, what effect did the boat's velocity have on the ball's vertical acceleration due to gravity? Why did the ball not fall behind the mast?
Q2 In watching Event 2, from the boat's frame of reference the object may seem to have been pushed sideways relative to the boat. Is this a possible interpretation and, if so, what would apply the force?

Figure 4.2



Chapter 5. The Birth of Dynamics

Aristotle's theory required that to initiate or sustain *violent* motion, even if it were uniform, required a push or a pull. Is this always true?

The following is really a thought-experiment in which you will examine what appears to be some very simple phenomena and try to explain what happens without taking measurements. The questions presented here, along with those which occur to you are very important and should be answered carefully as you are doing the experiment.

Experiment 5.1 Inertia

Place a steel ball on a ruler or track attached to a cart as shown in Figure 5.1.

First, with the ball at the centre of the track, give the cart a quick push across a level desk so that it accelerates rapidly.

Q1 Describe the ball's motion during the cart's acceleration. (This may be ambiguous if you realize that you can describe the motion of the ball relative to the cart or relative to you the observer.) Try it again and describe the ball's motion *relative to you.*
Q2 If the ball had been taped to the cart, instead of being left free to roll (there is very little friction between the ball and track) how would it have behaved? Why?
Q3 Try to account for the difference in your answers to Q1 and Q2.
Q4 Based on your observations, what is the most "natural" behaviour for an object at rest? Give some other examples of this "natural" behaviour.

Next, get the cart moving at uniform motion with the ball staying at the centre of the track. Then pull back on the cart so that it decelerates rapidly.

Q5 Describe the ball's motion relative to you as the cart decelerates.
Q6 During the cart's uniform motion, is the ball's motion *natural* or *violent*
(a) in terms of Aristotle's Theory of Motion?
(b) as far as you are concerned?

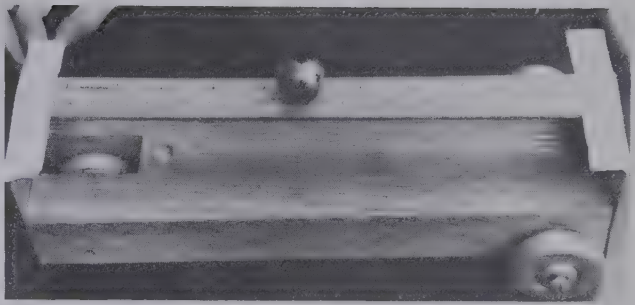


Figure 5.1

Q7 Is a force necessary to sustain uniform motion?
Q8 Describe the motion of the ball *relative to you* as the cart decelerated (before the ball hits the end of the cart). Why did the ball behave in this way?
Q9 Was there a force which sustained the ball's motion during the cart's deceleration?
Q10 What is the most *natural* motion for an object already moving with constant speed?
Q11 How would your results compare if you had used a larger ball in the experiment? Why?
Q12 What would happen to the ball on a cart with a flat top, moving at constant velocity, if the cart made a left turn? Why?

The results here may seem obvious, but it took man almost two thousand years to conclude that it is as "natural" for an object to maintain constant velocity as it is for it to stay at rest if no other object acts on it. The states of rest and of uniform motion are dynamically equivalent.

Q13 Use an example to show the dynamic equivalence of the states of rest and uniform motion.

CHANGING THE FRAME OF REFERENCE

If you wish to think about this some more, try to imagine yourself as an observer on the cart. Review the description of the motion of the ball as seen by you, the observer, on the cart during (a) acceleration of the cart, (b) uniform velocity of the cart, and (c) deceleration of the cart.

Q14 From this new frame of reference would the ball appear to obey the Law of Inertia? Try to account for the ball's motion as seen from the cart's frame of reference in each case.

AN IDEALIZATION

You will remember that in Experiment 2.1 Uniform Motion, it was impossible to obtain perfectly uniform motion of an unpowered object here on earth, because of friction. Consequently, you might conclude that since uniform motion is impossible here on earth, then Newton's First Law is invalid. But if we could eliminate friction we would see uniform motion at least as nearly as we could tell. During space flights, man has come very close to actually seeing an unpowered object maintain constant velocity.

Q15 Describe the best possible situation for observing uniform motion and the operation of Newton's First Law for a non-powered object.

Q16 Suppose that an object were launched into space so that it could travel out among the stars and perhaps even out of our galaxy. In its trip would it always behave according to Newton's First Law of Motion? Would it travel in a perfectly straight line? Discuss.

Experiment 5.2 Newton's Second Law

Man has sought to understand the cause of observed changes in motion of any object. As part of the evolution of this understanding, Newton spelled out his second Law of Motion. Newton called the cause of a change in an object's state of rest or of uniform motion in a straight line, a *force*. His Second Law states that this force, when applied to an object, varies directly with the product of the mass and acceleration of the object. Also, the force and acceleration will be in the same direction. Mass and acceleration can be measured, and Newton's Second Law defines force in terms of these two measurable quantities, in the statement $F = ma$.

In doing this experiment you will get a feeling for the behaviour of F, m, and a, and also get some experience with the units used to measure these quantities. You can extend the experiment to consider the effects of uncertainty in your measurements if you wish, but this is optional.

The basic setup to be used is shown in Figure 5.2.

You will measure forces by the amount of stretch of a spring scale. Try to get one calibrated in newtons. (This unit is defined in your text.) If your spring scale uses some other units, your teacher can show you how to convert your force measurements to newtons.

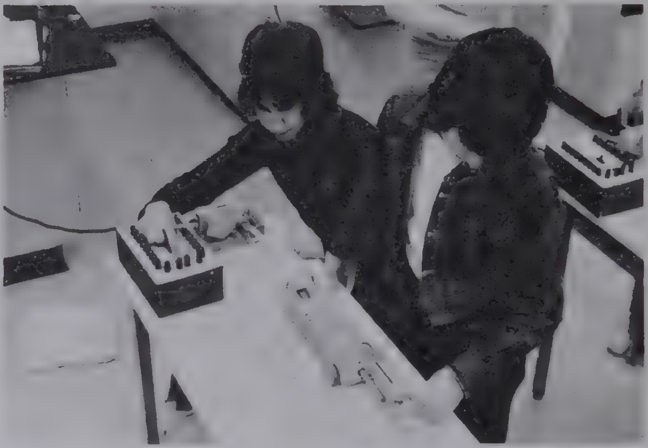


Figure 5.2

ACCELERATION WITH A CONSTANT FORCE

First investigate how an object accelerates when a constant force is applied. Begin with a force of 1 newton and practise pulling the cart until you can keep the force fairly constant over a distance of 1 to 2 metres. Then, to see how the acceleration behaves, use a ticker tape and vibrating timer or strobe photograph to determine the distance travelled in equal time-intervals.

From your observations, plot a speed-time graph.

Q1 Is the acceleration of the cart constant when the applied force is constant?

TESTING NEWTON'S SECOND LAW

Use various values of force F to pull the cart. Then vary the mass m, by piling other carts (or bricks) on the first cart. In each case measure the acceleration a.

To find the acceleration, we can use the fact that the acceleration is constant for a constant force. For uniform acceleration a, in a time-interval T, the distance travelled by an object starting from rest is,

$$d = \frac{1}{2}aT^2 .$$

Therefore, to measure a you need only to measure the time for the uniformly accelerated cart to travel a measured distance. Mark off a convenient distance (2 metres, or as long as possible if less than that) on the table. For each run, measure the time for the front of

Table 5.1

	Force F (N)	Mass m (kg)	Distance d ( )	Time T (s)	Acceleration a (m/s^2)	$m \times a$ ( )
same mass	1	1				
	2	1				
	3	1				
	⋮	⋮				
same force	1	2				
	1	3				
	1	4				
	⋮	⋮				

the cart to accelerate from the start to the finish, being sure it starts from rest and the applied force is as constant as you can make it.

Record your data for each run in a table similar to Table 5.1.

Analysis

Calculate the values of the product of $m \times a$ for each case.

Q2 What are the units of $m \times a$?

Q3 Do your observations support Newton's Second Law? Discuss.

Q4 What are the sources of error in your results?

Q5 What is the *net force* acting on the cart if you take friction into account?

Q6 What effect would friction have on the acceleration in each case?

Q7 Would an increase in the mass affect the friction on the cart? Discuss.

Q8 Based on your results, is there reason to believe that friction had a measurable effect on your results? In which case would such an effect be most noticeable? Discuss.

Experimental Uncertainty

In this experiment you are comparing two quantities F and $m \times a$ which should be equal according to Newton's Second Law. However, it is impossible to measure any physical quantities such as distance, time, mass, or force with absolute precision or certainty. Consequently, it is likely that F and $m \times a$ will not be exactly constant no matter how carefully you perform the experiment. This does not mean that Newton's Law is wrong or that you did the experiment incorrectly. If there is a large amount of uncertainty in your measurements, you cannot expect F and $m \times a$ to agree closely even though Newton's Law is valid. If, on the other hand, the uncertainty in your measurements is small, then F and $m \times a$ must agree much more closely.

In any experiment, to decide whether or not the results are valid, we shall use the following rule.

If the results of an experiment and the expected or accepted result agree within the limits of uncertainty, we can conclude that the results are valid. If they do not agree, then either the experiment was done incorrectly or the theory is wrong.

Before doing the experiment, consider the following. As you do the experiment, record the uncertainty of each of your measurements. Then, using the examples as a guideline, determine the limits of uncertainty for your experiment. (See Appendix C for more information on measurement and uncertainty.)

Uncertainty in F.

The force varied slightly as you pulled the cart. For example, it might have ranged from 6.8 N to 7.2 N as you attempted to maintain a constant force of 7.0 N. You then record F in the table as (7.0 ± 0.2) N. In terms of percentage uncertainty this would be 7.0 N ± 3%.

Uncertainty in m.

Uncertainty in m depends on the balance used. If its scale is marked to the nearest one-tenth of a kilogram, then your measurement will have a range of uncertainty of plus or minus one scale division, or in this case ± 0.1 kg. If the total mass being towed were 2.0 kg, then your value of m could be recorded as (2.0 ± 0.1) kg, or in terms of percentage uncertainty, $m = 2.0\ \text{kg} \pm 5\%$.

Uncertainty in a.

Since you used the equation $a = 2d/T^2$, the uncertainty in the calculated value of a will depend on the uncertainties in the measured values of d and T. The uncertainty in d depends on your measuring instrument and on the technique used. Suppose you could measure d to the nearest mm. Then d might be (2.000 ± 0.001) m, or 2.000 m ± 0.05%. The uncertainty in T depends on your reaction time and the accuracy of the stopwatch. Suppose the stopwatch is accurate to the nearest 0.05 s and that your reaction time is also 0.05 s. The total uncertainty is ± 0.1 s and a possible value of T would be (1.0 ± 0.1) s or 1.0 s ± 10%. The uncertainty of T^2 is double the uncertainty of T so that, in this case, $T^2 = 1.0\ \text{s}^2 \pm 20\%$.

To find the range of uncertainty of a, we use the rule that the uncertainty of a quotient is the sum of the percentage uncertainties of the two values. For the examples given,

$$a = \frac{2 \times (2.0\ \text{m} \pm 0.05\%)}{1.0\ \text{s}^2 \pm 20\%}$$

$$= 4.0\ \text{m/s}^2 \pm 20.05\%$$

$$\cong 4.0\ \text{m/s}^2 \pm 20\%,$$

as the uncertainty in d is negligible.

Uncertainty in ma.

The uncertainty of a product equals the sum of the percentage uncertainties in the factors, here in m and a.

Using the examples given,

$$ma = (2.0\ \text{kg} + 5\%)\cdot(4.0\ \text{m/s}^2 \pm 20\%)$$

$$= 8.0\ \text{kg m/s}^2 \pm 25\%$$

Comparing F and ma.

In the examples above the values of F and ma are,

$F = 7.0\ \text{N} \pm 3\%$ or $(7.0 \pm 0.2)\text{N}$

$ma = 8.0\ \text{kg m/s}^2 \pm 25\%$ or $(8.0 \pm 2.0)\ \text{kg m/s}^2$

Based on the uncertainties in the example, the force F, which could be as low as 6.8 N or as high as 7.2 N in this case, still lies between the limits of the uncertainty in ma which could be as low as 6.0 N or as high as 10.0 N. Therefore we can conclude that $F = ma$ within the limits of experimental uncertainty.

Now look at your results and in the same way as shown above, determine if your results are valid.

Experiment 5.3 An Explosion

When two objects explode apart, both of them will accelerate if they are free to move. Is there a general principle which can be used to predict the effect of one object on another in such an interaction?

In this experiment you will study an explosion between two carts A and B. One is loaded with a spring which can explode them apart. The masses of the carts can be changed in effect by piling more carts on either side.

It is not necessary to measure the actual accelerations in order to understand what is happening. Instead, you will determine the ratio of the accelerations of A and B and see how they are related to the ratio of the

Figure 5.3

masses of A and B. It is not even necessary to measure the masses of A and B in kg. Instead use the mass of one cart as your unit of mass.

You can compare the motion of A and B simply by measuring the distance travelled by A and B in the same time. Clamp two bumpers across the ends of the table. Adjust the launching point of A and B until both carts hit their respective bumper at the same time. Measure the distances d_A and d_B as shown in Figure 5.4.

Procedure

Set up two similar carts at the centre of a level table and compress their springs. Place them together so that when the launching pin of one is struck, its spring will release and push the carts apart. Before launching, predict what will happen to A and B. Then try it and record your measurements of d_A and d_B in a table similar to Table 5.2.

Now study the effect of changing the masses of A and B at least in the cases shown in Table 5.2. In each case, before launching, predict what will happen. Adjust the launch point and the distances, according to your prediction, and then experiment until the carts hit the bumpers simultaneously.

Compare your predictions and the experimental results.

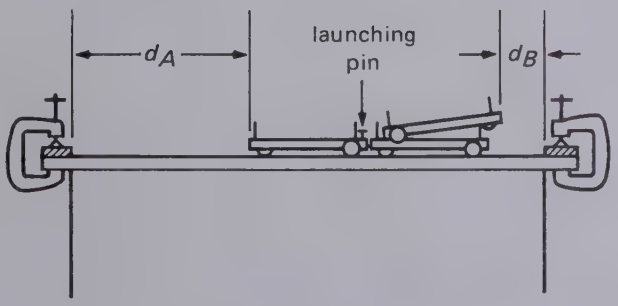


Figure 5.4

Table 5.2

m_A (cart)	m_B (cart)	d_A (m)	d_B (m)	$\frac{d_A}{d_B}$	$\frac{a_A}{a_B}$	$\frac{m_A}{m_B}$	$\frac{F_A}{F_B}$
1	1						
1	2						
2	1						
2	2						
3	1						

Analysis

By definition, $v = d/\Delta t$ and the ratio of launch speeds of A and B can be written,

$$\frac{v_A}{v_B} = \frac{\frac{d_A}{\Delta t_A}}{\frac{d_B}{\Delta t_B}}$$

In the experiment, the times Δt_A and Δt_B are the same, so that,

$$\frac{v_A}{v_B} = \frac{d_A}{d_B}$$

Similarly,

$$a = \frac{\Delta v}{\Delta t},$$

and the ratio of accelerations,

$$\frac{a_A}{a_B} = \frac{\frac{\Delta v_A}{\Delta t_A}}{\frac{\Delta v_B}{\Delta t_B}}.$$

Since the carts start from rest,

$$\Delta v_A = v_A \text{ and } \Delta v_B = v_B.$$

Also the times are equal so that,

$$\frac{a_A}{a_B} = \frac{v_A}{v_B} = \frac{d_A}{d_B}.$$

Therefore, the ratio of accelerations which we wanted is the same as the ratio of distances in the experiment.

Calculate the ratios $\frac{d_A}{d_B}$, $\frac{a_A}{a_B}$, and $\frac{m_A}{m_B}$ and tabulate the results.

Examine your results and state a general rule relating the masses and their accelerations.

Compare the directions of the accelerations in all cases.

Q1 What would happen if one of the masses were made very large? infinitely large?

Q2 Suppose the carts were travelling along together at the same speed when the explosion occurred.

(a) Describe what would happen.

(b) Would the same relationship hold as shown in this experiment?

FORCES

Newton explained changes in motion in terms of forces. In his Second Law he stated that $F = ma$. You can use this definition to compare the forces acting on A and B in the experiment. The ratio of forces,

$$\frac{F_A}{F_B} = \frac{m_A \cdot a_A}{m_B \cdot a_B}.$$

Or, in terms of mass and acceleration, ratio

$$\frac{F_A}{F_B} = \frac{m_A}{m_B} \cdot \frac{a_A}{a_B}.$$

Using the ratios in Table 5.2 find the ratio of the forces for each case.

Q3 Within the limits of experimental error, do your results confirm Newton's Third Law of Motion? Discuss.

Experiment 5.4 Balancing Forces

The Aristotelian concept of force can be summed up in the phrase, a force is a push or a pull. We usually go on to think that if a force is applied to an object, its motion will change. It will speed up, slow down, or be pushed in another direction. But objects at rest also have forces acting on them. For example, in a tug of war between two equally matched teams, if the forces exerted on the ends of the rope by each team balance one another, the rope does not move. In this experiment you will study forces which balance each other, or are *in equilibrium*. Also you will investigate the condition necessary for this equilibrium.

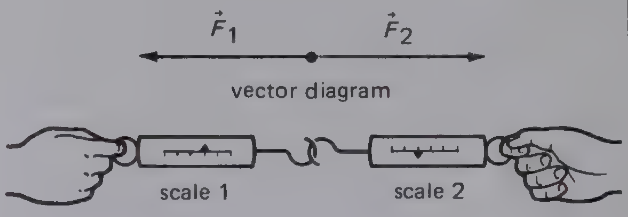


Figure 5.5

Procedure A Forces in a Straight Line

Obtain two similar spring scales. Hook the scales together and pull on each in opposite directions as shown in Figure 5.5. Record the force exerted by each spring scale on the point where they are connected.

Draw a force-vector diagram to scale to illustrate your results.

Find the sum of the two forces $\vec{F}_1$ and $\vec{F}_2$ by adding the vectors tip to tail.

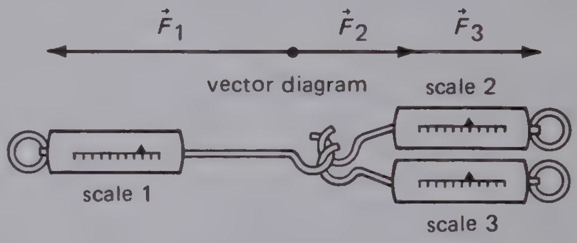


Figure 5.6

Place another spring scale parallel to one of the others as shown in Figure 5.6.

Repeat the experiment measuring the three forces. Draw a vector diagram showing $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$ and find the vector sum of the forces.

Q1 What should be the vector sum of forces which balance one another or are in equilibrium? Do your experimental results confirm this? Discuss.

Procedure B Forces in a Plane

Obtain 2 C-clamps, a piece of string, and a piece of polar graph paper or a protractor. You will need three persons in your group, one for each spring scale. Attach two of the scale hooks with the piece of string and set up on a table as shown in Figure 5.7.

Adjust the C-clamps and the length of string so that there is a force of between 1 and 5 newtons on each scale.

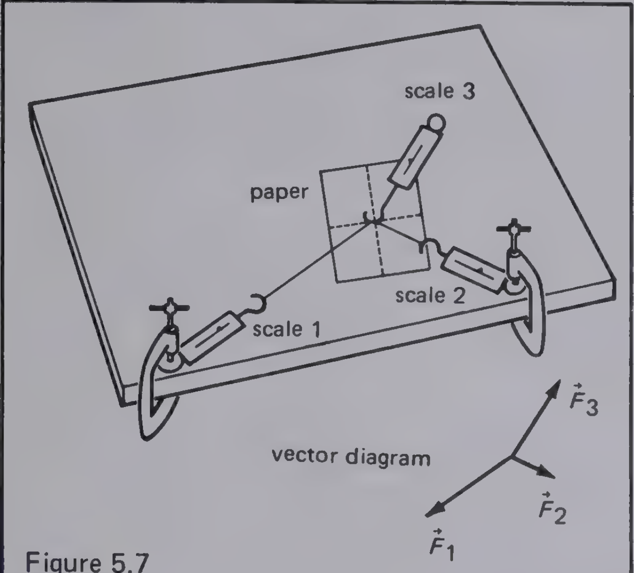


Figure 5.7

Place a piece of paper beneath the intersection of the forces and carefully mark the direction of each of the three forces with a line segment drawn from the intersection point. This line segment will form the basis of a vector diagram of the forces. Also, record the magnitude of each force on your paper. If you use polar graph paper, read the direction of each force directly.

Read just the tensions and the angles between the forces and repeat the experiment so that each person in your group has a separate case to analyse.

Analysis

Establish a scale for the force vectors so that they can be drawn on the page and so that the diagram is not too small. For example, if one force is 4.0 newtons, you could make that vector 8.0 cm long.

For each case find the vector sum from the diagram of two of the forces, $\vec{F}_1$ and $\vec{F}_2$. This sum is the resultant force of $\vec{F}_1$ and $\vec{F}_2$. (Newton discovered that the effect of the resultant force on an object was the same as having the two separate forces act simultaneously.)

Q2 From your diagram, what can you say about the relationship between the resultant force-vector ($\vec{F}_1 + \vec{F}_2$) and the third force-vector, $\vec{F}_3$?

Test the relationship by finding the resultant of $\vec{F}_1$ and $\vec{F}_3$, and comparing it with $\vec{F}_2$.

Q3 Would you get the same results if you added the magnitudes of the forces? Why?

Now add all three forces.

Q4 Based on your diagram, what can you say about the total sum or resultant of three forces in equilibrium?

Additional Questions

Q5 Would the same results be obtained if more than three forces acted on a point? If you have time, you can experiment by obtaining another spring scale.

Activities

"You can't push a rope."
Author unknown

"The whole burden of philosophy seems to consist in this — from the phenomena of motion to investigate the forces of nature, and then from these forces to explain the other phenomena."
Sir Isaac Newton (1642-1727)

WHY ARE YOU DRAGGING THAT CHAIN AROUND?
DID YOU EVER TRY PUSHING ONE?
hart

Activity 5.1 Inertia Demonstration 1

Place a dime on a smooth card on a water glass as shown.

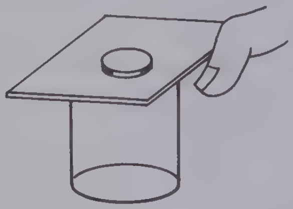


Figure 5.8

With your finger give the card a sharp snap. Describe and explain what happens. Replace the card with a piece of sandpaper, rough side up, and repeat the performance. Why was the behaviour not the same?

Q1 What are the differences between friction and inertia?

An alternate version is to pour some water in a drinking glass — not too full — and place the glass on a piece of paper at the edge of a desk.

Pull the paper out from under the glass.

Q2 Why should you pull it quickly?

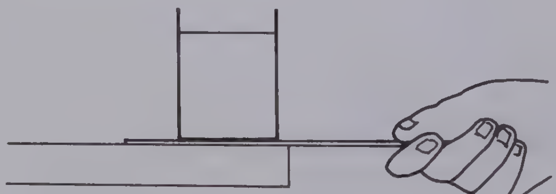


Figure 5.9

Activity 5.2 Inertia Demonstration 2

If you can obtain a sledge hammer head and a separate handle, try to place the head on the handle in two ways.

1. Head at rest. Have someone hold the head in mid-air. Then drive the handle onto the head with a mallet, as shown in Figure 5.10.

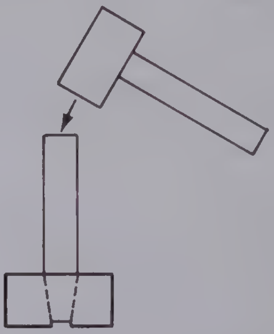


Figure 5.10

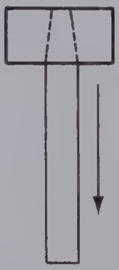


Figure 5.11

2. Head moving. Strike the handle on the floor to drive the head on the handle as shown in Figure 5.11.

Explain what happens in each case, using Newton's Law of Inertia.

Activity 5.3 Inertia Demonstration 3

Stack several blocks of wood on a table then take a "swipe" at one of the blocks with a metre stick. Try the one at the bottom and, if you can get someone to catch it, one in the middle of the pile.

Activity 5.4 Inertia Demonstration 4

Hang a weight (some labs have an inertia ball for this demonstration) with a light string from a strong support. Attach a second string to the bottom of the weight. Place a cushion directly below the weight.

Give the bottom string a quick hard jerk. Describe and explain what happens. Then repeat, but this time, pull on the string slowly and steadily.

Activity 5.5 Ballistics Cart

If a ballistics cart is available, study its behaviour. It is spring-loaded to launch a ball vertically when the cart is moving.

Secure the string attached to the launching pin to one end of a desk. Adjust the length of the string so that it will "yank" the pin shortly after the cart is started. Then give the cart a strong push. Have someone at the other end of the table to catch the cart, and the ball too if it doesn't fall back into the launching tower. Describe and explain what happens.

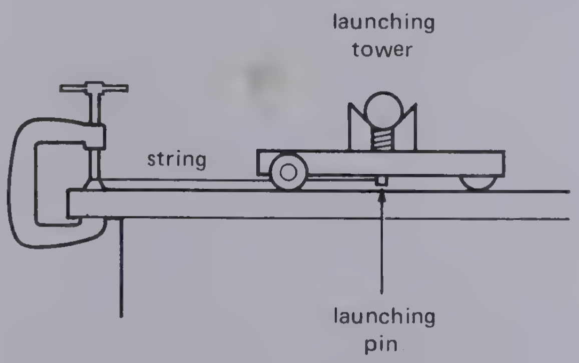


Figure 5.12

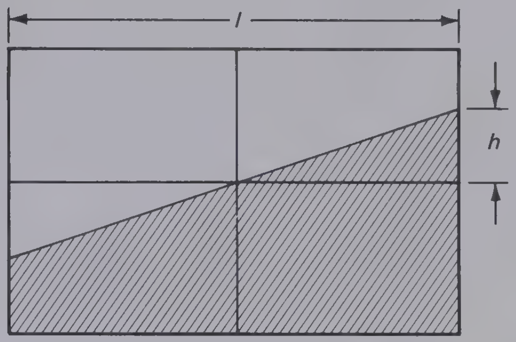


Figure 5.13

Activity 5.6 Accelerometer

LIQUID-SURFACE ACCELEROMETER

The accelerometer is a thin vertical tank partly filled with a coloured liquid which can be used to indicate acceleration. You can use this accelerometer to measure actual values of the acceleration.

Attach the accelerometer securely to an object which is to be accelerated, such as a collision cart.

When the cart is at rest, the liquid surface is horizontal. Try accelerating, decelerating, and moving the cart at uniform velocity. Describe what happens in each case.

MEASURING ACCELERATION

The slope of the liquid surface increases as the cart's acceleration does. When the cart is accelerated the surface will be tilted so that one end is height h above its rest position as shown in Figure 5.13.The length of the accelerometer is l.

The theory shown in the box gives a simple relation between the acceleration, h, and the length, l.

$$a = \frac{(2h)}{l} a_g.$$

In order to determine the acceleration of an object to which the accelerometer is attached, all you need to measure during the run is h. This is difficult to do visually, but you can repeat the run several times and take an average of your readings. Another method is to take a photograph of the accelerometer during the run and then measure h from the photograph.

ANOTHER LIQUID-SURFACE ACCELEROMETER

This form of liquid-surface accelerometer can be easily made by bending a piece of glass tubing and partially filling it with the coloured water as shown in Figure 5.14.

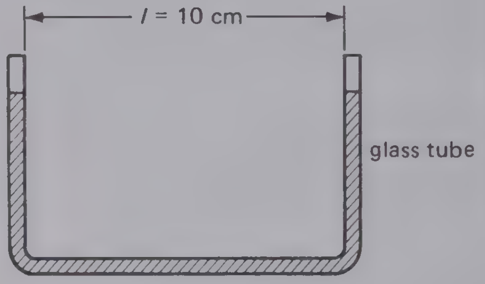


Figure 5.14

This accelerometer works according to the same theory as the previous one. A scale can be taped on one side of the tube for measurement of h directly. Then determine the acceleration a using,

$$a = \frac{2h}{l} \cdot a_g .$$

The accelerometer is easily portable and could be experimented with in a car, by a passenger and not the driver of course!

CORK-IN-BOTTLE ACCELEROMETER

Attach a cork with a string to the lid of a jar. Fill the jar with water, insert the cork and string, tighten the lid and invert the jar. It may be necessary to use some sealer to avoid leakage.

Now accelerate the jar. The behaviour of this accelerometer may surprise you. Try to explain its behaviour.

Activity 5.7 Design Your own Accelerometer

In Activity 5.6, only three kinds of accelerometers are shown. You can design your own accelerometer based on the same principle but using completely different components.

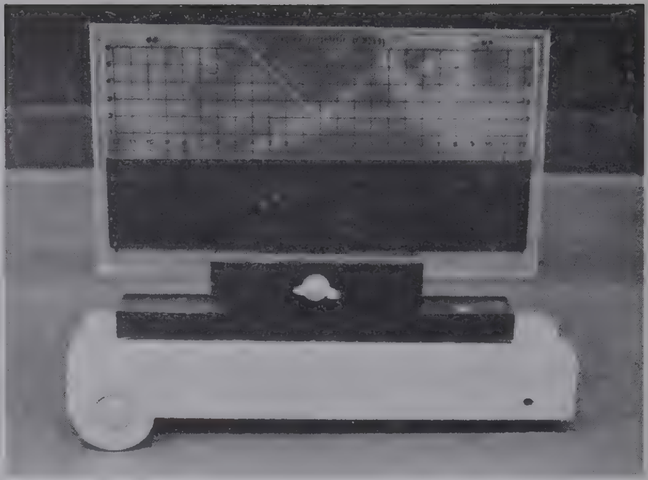


Figure 5.15

Activity 5.8 Free Fall and Weightlessness

Tie a weight to the hook of a spring balance, with a large easy-to-read scale. Hold up the balance to determine the weight of the object.

Drop the balance and the object from the ceiling onto a thick sponge pad on the floor and watch the reading on the balance as it falls toward the floor.

Q1 How should the acceleration of the weight and balance compare? Why?

Q2 Why did the reading on the balance become zero, or almost zero, during free fall?

Q3 Did the object become weightless during free fall? Discuss.

Q4 What would the balance read if it and the weight were

(a) falling at constant velocity?

(b) accelerating upward at 9.8 m/s^2?

Activity 5.9 An Action-Reaction Car

You may have noticed the vehicle designed by Newton shown on page 86 in the text. An interesting and challenging activity is to build a working model based on the same principle as the one shown. When you have built the model, explain clearly how it works.

If several members of the class build vehicles, arrange a race with the help of your teacher. Collect a small entry fee for each vehicle and use the fees for prizes.

Theory for a Liquid-Surface Accelerometer

As the cart accelerates the liquid forms a flat surface at an angle θ to the direction of acceleration. As the acceleration increases, θ also increases. If you consider a small element of mass of the liquid on the surface and consider the forces on this element m, you can derive a relationship betwen a and θ.

The forces on m are:

1. The force of gravity $m\vec{a}_g$ acting downward,
2. The reaction force of the liquid, $\vec{R}$. This force must act perpendicular to the surface as all elements of the liquid on the surface are free to move parallel to the surface. This means that no component of this liquid-on-liquid reaction-force can exist parallel to the surface.

These two forces combine to provide the accelerating force on the element m. Since the liquid moves along at the same acceleration as the cart, the resultant of R and $m\vec{a}_g$ must be $m\vec{a}$ in the direction of the acceleration.

In the triangle of forces, the angle between $\vec{R}$ and $m\vec{a}_g$ is θ. Therefore,

$$\frac{ma}{ma_g} = \tan\theta.$$

Dividing m into m, $\dfrac{a}{a_g} = \tan\theta.$ (1)

The acceleration can be determined by measuring θ and substituting in the equation (1) above. It may be more convenient to measure the *slope* of the surface.

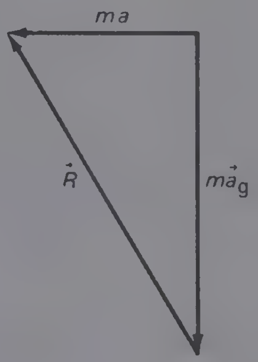


Figure 5.17

If the length of the accelerometer is l and the height of the high end of the surface above its rest position is h, then,

$$\text{slope} = \frac{h}{\frac{1}{2}l}.$$

In terms of θ,

$$\tan\theta = \frac{h}{\frac{1}{2}l}. \qquad (2)$$

Combining equations (1) and (2),

$$\frac{a}{a_g} = \frac{h}{\frac{1}{2}l}$$

$$a = \frac{2h}{l}\,a_g. \qquad (3)$$

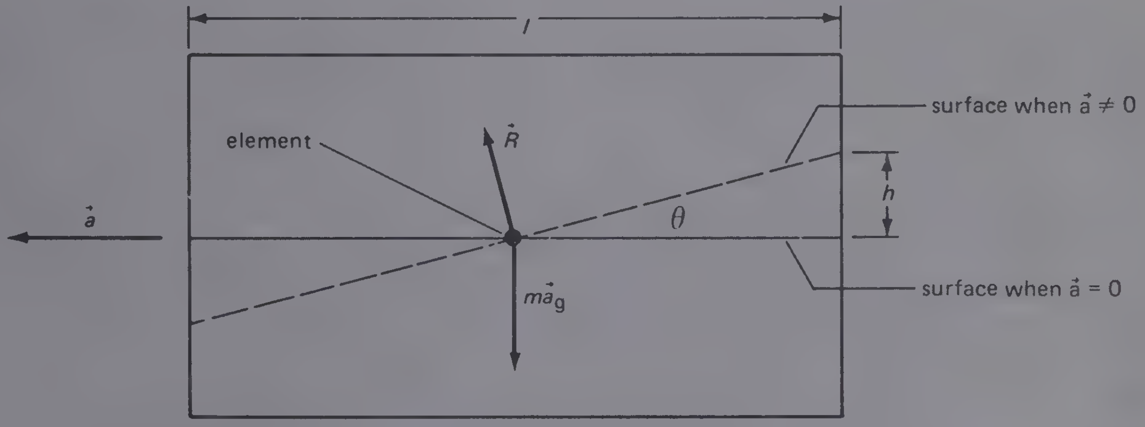


Figure 5.16

Appendix A Photography

In several experiments, the use of photography is recommended as a possible method of recording observations. This appendix outlines the steps to follow in taking a Polaroid photograph in the laboratory. Some of the hints will be helpful also, if you wish to use another type of camera.

Film. In most experiments use black and white 3000 ASA film. Make sure the camera contains film before you start. If there is film in the Polaroid camera, a white tab will show in front of the flap marked (4). The number on this tab indicates the number of the exposure. (There are 8 exposures for each pack of Polaroid film.) If there is no film in the camera, open the back of the camera by flicking the switch inset on the side of the camera and place the film pack in as shown in the instructions accompanying the film.

Set up. Fasten the camera to a tripod or set it on a solid support such as a table, at the same level as the object to be photographed. The distance from camera to object should be between 1 and 2 metres depending on the field of view required. For easy measurement it is a good idea to work at a distance for which the picture is one-tenth actual size. Another suggestion is to place a reference metre stick in the picture so that the scale of the photograph can be determined.

Electric Eye. For the time-exposure ("bulb" exposure) cover the electric eye if your model has one. Keep this eye covered when the camera is not in use so that the batteries in the camera are conserved.

Exposure. Set to the *B* setting for time-exposure photographs. The shutter will stay open as long as shutter release button is depressed for this setting.

Focus. Look through the viewer to be sure that the entire experiment is within the field of view outlined by the yellow border in the viewer. Adjust the focus until there is a single image of the object in the yellow square at the middle of the viewer.

Exposure. The amount of light entering the camera is controlled by the exposure time and the opening of the diaphragm. The exposure time is determined by the experiment. Adjust the diaphragm opening by changing the EV number, if this setting is available on your camera, to give a properly exposed picture. If the first attempt is over-exposed (too white) increase the EV number. If underexposed, decrease the EV number.

Dry Run. Try the experiment once or twice without taking a photograph so that you can synchronize your operation of the shutter release with the beginning and ending of the experiment.

Shutter Release. Set the shutter by pushing the lever by the lens (marked (3)), all the way down.

Taking the Picture. Don't move the camera. To eliminate unwanted light, keep the shutter release (red button marked (2)) depressed only as long as necessary to take the picture.

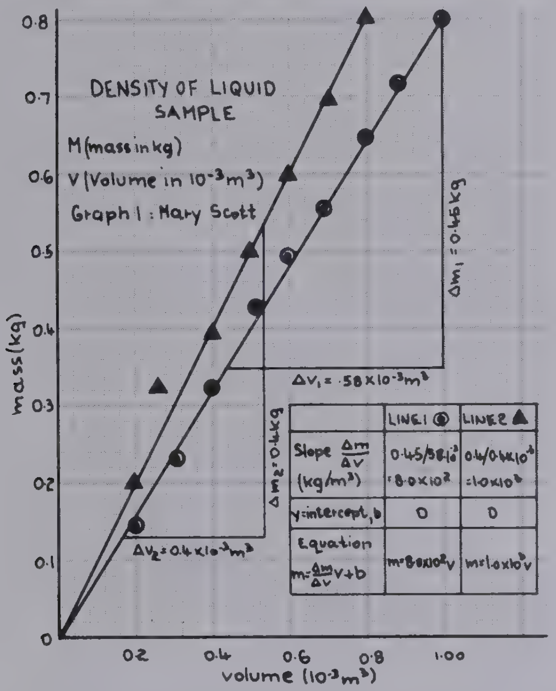
Developing. Pull out the white tab completely. Then pull the yellow tab steadily and straight out of the camera. Wait for 10 to 15 seconds. Separate the white print from the negative. Discard the messy paper around the border of the negative, but keep the negative as a second copy. Wash the negative with a wet sponge. Coat the print with the coater which comes with the film.

Measurement. The photo is too small to take direct accurate measurements with a ruler. One method of improving accuracy is to use a magnifying lens with a scale either on the lens or magnified alongside the images on the photograph. Another method is to punch tiny holes with a pin at each object loaction and then place the photograph on an overhead projector. The image projected on a screen or chalkboard will be magnified. The magnification can be adjusted by moving the projector closer to or farther from the screen or chalkboard. If you have placed a reference length such as a metre stick in the photo, you can scale the photo to *actual* size by adjusting the magnification. Tabulate your measurements in a table similar to Table 2.1.

For the strobe or blinky you can use as your time unit, the time between successive images, and perhaps call it *1 flash* or *1 blink*. If you want to work in seconds ask your teacher how to calibrate the strobe or blinky.

Appendix B Graphical Techniques

Often relationships between data from an experiment become much clearer if plotted on a graph, than if left in a table of values. This appendix lists some important rules to follow in plotting a graph. These rules are illustrated in the graph which follows.



Axes. Each axis corresponds to a variable quantity obtained from the experiment. The independent variable, the one which you control, such as the time-interval between flashes of a stroboscope, is usually plotted on the horizontal axis. The dependent variable, which depends on values of the independent variable, such as the position of an object at particular times, is plotted on the vertical axis.

Draw the axes with a ruler.

Label the quantity being represented near each axis.

Choose a scale for each quantity so that as much of the sheet of graph paper is used as possible. A graph which is too small limits your ability to interpret its meaning. In most cases each scale should begin at zero. Each scale must be consistent so that each scale division corresponds to a constant interval. However, the scale divisions on the vertical scale do not need to correspond to the same numerical interval as those on the horizontal scale. This is illustrated in the graph which follows.

Plotting. Show each plotted point clearly. If more than one set of data is being plotted on the same axes, differentiate between points of each set by enclosing the points for one line with a circle, ⊙, or perhaps a square ⊡, or triangle ◬.

Drawing the Curve. Do not join the dots directly in a zigzag fashion. Instead draw the straight line or curve of "best fit". This is justified, as the measurements for all of the points are uncertain to some extent. It is impossible to find the equation for a "join the dots" line. Its shape is too complex. Nature does not behave in this way but rather in the way shown by the "best fit" line.

Title. Indicate in the title of the graph, the title of the experiment, the quantities plotted with their units, the number of the graph, and also your name if the graph is separate from the rest of your report.

Analysis of a Linear Graph. A straight line or linear graph is simplest to analyse. The straight line indicates that the relationship between the variables, say x and y, is represented by a function of the form

$$y = mx + b$$

where y is the dependent variable,
x is the independent variable,
m is the slope of the graph,
b is the y-intercept of the graph, (the value of y corresponding to the case where $x = 0$ or the point where the graph crosses the y-axis).

Slope. Slope is defined as $\Delta y/\Delta x$ where y and x are the dependent and independent variables respectively. To find the slope of a linear graph, choose any two convenient and well-separated points on the line of "best fit" and measure Δy and Δx (Δm and Δv as shown in the example). Unless the units of y and x are the same, the calculated slope will have units. The units of the slope will be the units of y divided by the units of x. The slope is useful in defining the relationship between the variables as shown above.

Appendix C Measurement and Uncertainty

All measurements are uncertain to some extent. Because of error in techniques and inaccuracy of instruments as well as fluctuations in the quantity being measured, it is impossible to obtain any measurement which is absolutely precise. Scientists recognize this and record their results showing the limits of uncertainty for each measurement. This appendix sets out some rules which are commonly used in recording data and calculating results.

Uncertainty in a Measurement. Suppose you were measuring the width of this sheet of paper with a metre stick marked off in millimetres. You could probably measure to the nearest millimetre considering the limitation of the scale and other sources of error such as parallax. If the measurement was 21.7 cm you would know that it was neither as low as 21.6 cm nor as high as 21.8 cm. In this case the range of uncertainty is ± 0.1 cm and you would record the measurement as 21.7 ± 0.1 cm.

For single measurements where uncertainty is due to limitations of the instrument scale, use as your range of uncertainty plus or minus one scale division.

If several repeated measurements are used, you can determine the limits of uncertainty by using the deviations from the average of these measurements. For example, if the boiling point of water is measured four different times giving results:

Measurements	Deviation from Average
99.7°C	0.3
100.0°C	0.0
100.4°C	0.4
99.9°C	0.1

$$\text{Av.} = \frac{400.0}{4} = 100.0^\circ\text{C} \pm \text{uncertainty} \ \frac{0.8}{4} = 0.2$$

The uncertainty can be shown in two ways:
maximum uncertainity — in the example the maximum deviation from the average is 0.4°C so that the measurement could be recorded as 100.0°C ± 0.4°C, or,
average uncertainty — obtained by finding the average of the deviations. In the example the average of the deviations is 0.2 so that the measurement could be written 100.0 ± 0.2°C.

Significant Digits. *The degree of precision of a measurement can be shown by the number of measured or significant digits shown in the result.*

Example 1. The length of a room is measured to the nearest cm and the result is 6.42m with 3 significant digits. If the measurement were done crudely so that it was accurate only to the nearest 0.1 m it would be recorded as 6.4 m with only 2 significant digits.

Example 2. The population of Toronto is 2,100,000. It is difficult to tell how accurate this count is but it is probably to the nearest 100,000 so that there are only 2 significant digits. The zeros are merely placeholders and are not significant. If the measurement is correct to the nearest 10,000, this can be shown by drawing a line over the last significant zero, $2,1\bar{0}0,000$.

Example 3. The mass of a lunar rock sample is 0.506 kg. Here there are 3 significant digits. The zero to the left of the decimal is not significant whereas the zero to the right is measured and significant.

Example 4. The thickness of a pipe is 0.50 cm. Here there are 2 significant digits. The zero to the right of the decimal is significant because it has been measured and recorded.

Scientific Notation (Standard Form). Much of the ambiguity about significant digits can be removed by writing the number in scientific notation. To do this, *move the decimal (divide) so that only one non-zero digit is to the left of the decimal and multiply by a power of 10 to compensate for moving the decimal.*

Example 1. To express 4905 in scientific notation, move the decimal three places to the left and multiply by 10^3 to compensate.

$4905 = 4.905 \times 10^3$ (4 sig. digits)

Example 2. $0.0056 = 5.6 \times 10^{-3}$ (2 sig. digits)

Example 3. $31\bar{0}00 = 3.10 \times 10^4$ (3 sig. digits)

Example 4. Showing the population of Toronto as 2.1×10^6 removes all doubt about the number of significant digits.

Rounding. If we multiply 1.5 cm × 2.37 in order to calculate the area of a rectangle, we get 3.555 cm^2 which has 4 significant digits. The product appears to have greater precision than either measurement.

The product cannot be more precise than the measurements and so it must be rounded off to have the same number of significant digits as the least precise measurement.

In this case the product is rounded to 2 significant digits and is 3.6 cm^2.

Example 1. $$\frac{4.471\ \text{cm}}{0.015\ \text{second}} = 2.914\ \frac{\text{cm}}{\text{second}},$$

should be rounded to $$2.9\ \frac{\text{cm}}{\text{second}}.$$

To simplify the arithmetic, round off all factors before calculating.

Example 2. $$\frac{4.93 \times 6.2015}{0.49}$$ $$= \frac{4.9 \times 6.2}{0.49}.$$

Fractional and Percentage Uncertainty. In making two distance measurements with the same ruler, the results could be (5 ± 1) cm and (100 ± 1) cm. The absolute uncertainty (± 1 cm) is the same in both cases but if considered relative to the measurement the uncertainty is much greater in the 5 cm measurement. To show the relative importance of an uncertainty, it can be expressed as a percentage uncertainty of the measurement.

$$\text{Percentage uncertainty} = \frac{\text{absolute uncertainty}}{\text{measurement}} \times 100\%$$

For the first measurement the percentage uncertainty is $\frac{1}{5} \times 100\% = 20\%$. The measurement can be written as 5 cm ± 20%. The second measurement is 100 cm ± 1%.

Operations with Uncertainties.

1. *Addition and Subtraction*

The masses of two cartons are,

2.5 ± 0.1 kg
12.3 ± 0.2 kg.

The absolute uncertainties signify that the sum could be as high as,

$$\begin{array}{r} 2.6\ \text{kg} \\ \underline{12.5\ \text{kg}} \\ 15.1\ \text{kg} \end{array}$$

or as low as,

$$\begin{array}{r} 2.4\ \text{kg} \\ \underline{12.1\ \text{kg}} \\ 14.5\ \text{kg}. \end{array}$$

The correct result is probably between these two values, near 14.8 kg. But the absolute uncertainty in the sum as shown by the calculations is ± 0.3 kg above or below the most probable result. The result should be recorded at 14.8 ± 0.3 kg. We can summarize with the following rule.

The absolute uncertainty of the sum of two numbers is the sum of the absolute uncertainties. The rule for subtraction is similar.

2. *Multiplication and Division*

For these operations work with percentage uncertainties only. Suppose the length and width of a room were recorded as follows:

length = 4.5 ± 0.1 m = 4.5 m ± 2%
width = 2.4 ± 0.1 m = 2.4 m ± 4%.

The area of the room would be 4.5 m × 2.4 m = 10.8 m^2 ± some uncertainty. This should be rounded to the correct number of significant digits as 11 m^2 ± some uncertainty. To determine the uncertainty here we use the following rule.

When multiplying or dividing, the percentage uncertainty of the product (quotient) is equal to the sum of the percentage uncertainties of the factors.

The basis for this rule is not shown here but you can prove it for yourself as an additional exercise. One approach is to use a numerical example as done in the previous section for uncertainties in addition and subtraction.

In our example the result should be recorded showing uncertainty as 11 m ± 6%.

3. *Powers and Roots*

To square a number you multiply it by itself, therefore to find the uncertainty use the rule for multiplication.

Example 1. The side of a square is 1.6 ± 0.1 cm or 1.6 cm ± 6%.

The area of the square is,

$$\begin{aligned} & (1.6 \text{ cm} \pm 6\%)^2 \\ = {} & 1.6 \text{ cm} \pm 6\% \times 1.6 \text{ cm} \pm 6\% \\ = {} & 2.6 \text{ cm}^2 \pm 12\%. \end{aligned}$$

The uncertainty of the result could also be obtained by doubling the percentage uncertainty of the number to be squared. In finding the cube of a number, it can be seen that the percentage uncertainty of the result would be three times the percentage uncertainty of the number which is to be cubed. In general, for powers the rule is:

The percentage uncertainty of a power X^n is n times the percentage uncertainty in the number X.

Mathematically, taking the root of a number is equivalent to finding a power with a fractional index. For example $\sqrt{X} = X^{½}$. To find uncertainty for a root, use the same rule as above where *n* is a fraction. Therefore the percentage uncertainty of the square root of a number will be ½ of the percentage uncertainty of the number.

Example 2. The volume of a cube is 8.0 cm^3 ± 6%. Since the length of one side of the cube is $\sqrt[3]{\text{volume}}$, then the length of one side

$$\begin{aligned} & = \sqrt[3]{8.0 \text{ cm}^3 \pm 6}\% \\ & = 2.0 \text{ cm} \pm 2\%. \end{aligned}$$

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Answers to End of Section Questions

Chapter 1

Q2 Natural motion—all motions in which a body moved to its natural place.
No forces are required to produce this.
Violent motion—motions in which a body moved to places other than its natural place.
Forces are required to sustain the motion.

Q3 The sphere with 10 units weight. Since it weighs more, it must contain more "earth" element, and would have a greater tendency to return to its natural place.

Q4 i) All matter is composed of one of four natural elements—Earth, Air, Fire or Water, or some combination of them.
ii) Objects made of Earth element would fall to the ground because that is their "natural place".
iii) No

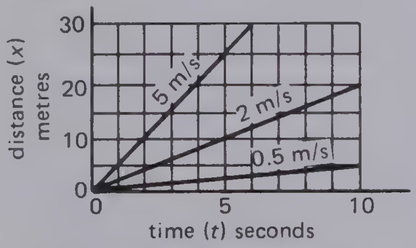
Chapter 2

Q1 Your answers should consist of a distance unit divided by a time unit

Q3 a) 6 cm/s
b) 15 km
c) 0.25 s
d) 24 m

Q4 a) 3 m/s
b) 20 m/hr
c) 1° C/s
d) 0
e) —75 dollars/day
f) 1 g/l

Q5 a)



Q6 a) 50 km, 100 km and 250 km
b) Uniform motion
c) 50 km/hr
d) The speed-time graph is an horizontal line cutting the vertical axis at $v = 50$ km/hr.

Q7 a) Object moving with uniform speed of 30 km/hr.
b) The area under the line.
c) Δx = area of rectangle = 30 km/hr × 3 hr
= 90 km

Q9 a) 6.7 m/s^2
b) 0
c) —10 m/s^2
d) 9.5 m/s^2
e) 0.5 km/hr/s

Q10 1.5 m/s, 10 s, Discussion

Q11 a) A accelerates at 5 m/s^2
B accelerates at 3 m/s^2
b) In 5 seconds A travels 62.5m
In 5 seconds B travels 37.5m

Q12 a) The initial speed is not zero.
b) 75 m

Q13 V average = 64 km/hr
Δt = 620 hours

Q14 V average = 22.5 m/s

Q15 a) An object initially travelling at a certain speed slows down and stops.
b) 20 m/s
c) 30 m/s, 10 m/s
d) Yes

Q16 a) 40 m/s
b) 46.5 m, 23 m/s
c) 1.67 m/s^2, 750 m
d) 3.3 s
e) 6.25 m/s^2, 25 m/s

Chapter 3

Q2 Galileo proposed that motion be called uniformly accelerated when there are equal changes of speed in equal time intervals.

Q4 Moving with uniform speed, a body covers equal distance intervals in equal times, whereas, in uniform acceleration, a body changes speed by equal amounts in equal times.

Q5 It is relatively easy to measure times taken to fall measured distances. Measuring instantaneous speeds is very much more difficult.

Q6 i) speed is constant
ii) acceleration is constant

Q7 e)

Q8 a), c) and d)

Q9 Strictly, only the rather narrow conclusion, that a body rolling down a slightly inclined ramp rolls down with a constant acceleration.

Q11 A body falling under conditions in which air resistance is zero or is so small that it can be ignored.

Q12 c)

Chapter 4

Q1 a) Hitchhiker moving south at 100 kilometres/hour
b) Other car at rest
c) Airplane flying south at 400 km/hr
d) Other car heading north at 1 km/hr

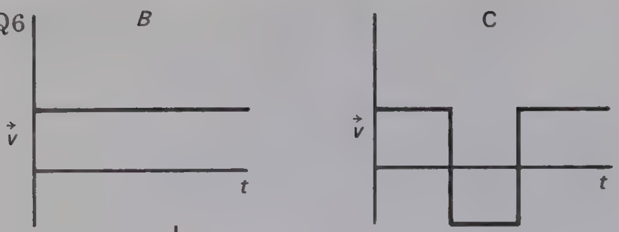
Q2 Distance and speed are scalar quantities.
Displacement and velocity are vector quantities.
Scalar quantities express a magnitude.
Vector quantities express both a magnitude and direction.

Q3 Distance travelled = 35 m
Displacement = 15 m to the left.
Average speed = 7 m/s
Average velocity = 3 m/s to the left

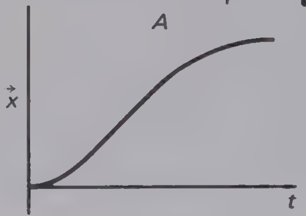
Q4 Average speed = 7.5 m/s
Average velocity = 0

Q5 Graph A: Object at rest accelerates uniformly, then travels at constant speed, slows down with uniform acceleration and stops.
Graph B: Object positioned some distance from reference point travels toward reference point at constant speed passes by and continues at constant speed in same direction.
Graph C: Object travels back and forth at constant speed.

Q6



Q7



Q8 a) $\vec{d} = -8T + \frac{1}{2}(+2)\,T^2$

$\vec{v} = -8 + (+2)\,T$

b)

t(s)	$\vec{v}$(m/s)	$\vec{d}$(m)
0	−8	0
1	−6	−7
2	−4	−12
3	−2	−15
4	0	−16
5	+2	−15
6	+4	−12
7	+6	−7
8	+8	0
9	+10	+9
10	+12	+20

c) Check values using values in charts.

Q9 $\vec{x} = 0$, $\vec{v} = +20$ m/s

Object located 30m to left at t = 22 and t = 6 s.

Q10 $\vec{x}$ = 18.3 cm 34° West of South

v = 3.7 cm/minute 34° West of South

Q11 a) 0.5 m/s in direction of wind

b) 1.5 m/s in direction of wind

c) 1.2 m/s at angle of 27° to wind direction

Q12 a) i) $\vec{d}$ = 12 km 20° E of N $\vec{x}$ = 12 km

ii) $\vec{d}$ = 12 km 20° E of N $\vec{x}$ = 48 km

b) i) $\vec{v}$ = 1.1 km/hr 20° E of N v = 1.1 km/hr

ii) $\vec{v}$ = 1.1 km/hr 20° E of N v = 40 km/hr

Chapter 5

Q1 Kinematics is the study and description of motion.

Dynamics is the study of the *causes* of motion.

Q2 Both

Q3 Vector quantities have both a magnitude and direction associated with them, whereas scalar quantities have magnitude only.

Q4 Case 1 1 unit [right]
Case 2 2 units [right]
Case 3 zero

Changing the sign convention makes no difference to the answer.

Q5 7 units [left]

Q6 Vectors may be added in any order. The tail of a new vector is attached to the head of the preceeding one. The resultant vector is always directed from the tail of the first to the head of the last.

Q7 The net force is that single force which would have the same effect on the motion of the body as do all the real forces acting on the body.

Q9 Aristotelian—constant velocity motion requires a constant force.

Newtonian—constant velocity motion requires no net force.

Q10 Inertia is the name given to describe the property that all objects exhibit of maintaining their velocities unchanged unless acted upon by an external unbalanced force.

Q11 Student examples

Q12 The same

Q13 An object that is acted on by an unbalanced force experiences an acceleration in the direction of the force. The acceleration varies directly as the force and inversely as the mass of the object.

$$a \propto \frac{F}{m}$$

Q14 False. The Force referred to in the second law is the net force. The force of friction may well be one of the real forces acting on the body. It will contribute to the value of the net force.

Q15 5 m/s^2, 3 m/s^2, 1.5 m/s^2

Q16 Zero

Q17 2.5 kg

Q18 45 N

Q19 Discussion

Q20 If object A exerts a force on object B, then object B exerts a force equal in magnitude but opposite in direction on object A.

Q21 Any one force and its reaction force, as defined by Newton's Third Law.

Q22 Action—reaction forces act on *different* bodies. The state of motion of any one body is determined by the net force acting on that body, not by forces acting on any *other* body.

Q23 Discussion

Q24 Yes. The force of gravity decreases by the same amount on both the standard kilogram and the unknown mass, therefore the balance remains "in balance".

Q25 c, f

Q26 e, f

Q27

Weight is a force	Mass is not a force
Weight of a body can change	Mass of a body does not change
Weight is measured with a spring balance	Mass is measured with an equal arm balance

Q28 Does not change at all

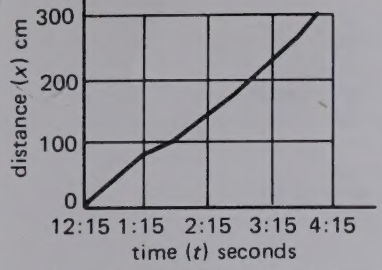
Q29 Zero

Answers to End of Chapter Problems

Chapter 2

2.1 1125 km
2.3 13.1 m
2.4 3650 km

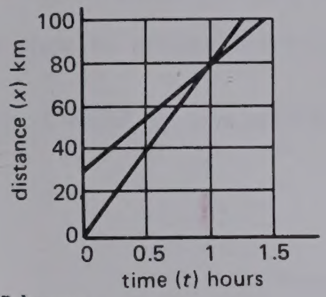
2.5 a) 80 km/hr
b) 4:15 pm
c) not known
d)



Average speed C — D 80 km/hr
Average speed D — E 90 km/hr

2.6 7.9 km/s

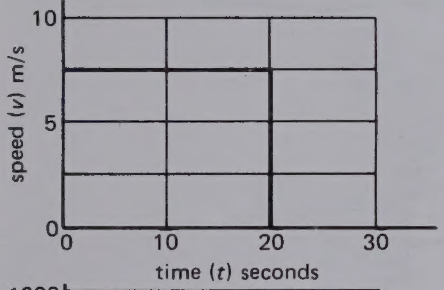
2.7 Car A overtakes car B after 1 hour.



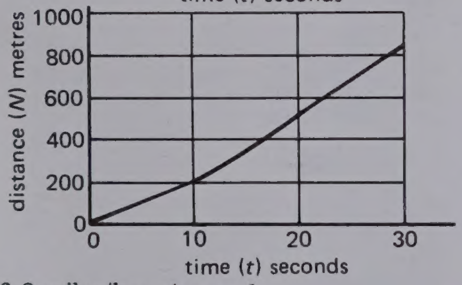
2.8 1.5 hours

2.9 a) 120 km/hr
b) 0.12 km/hr/s
c) 100 km
d) 91 km/hr

2.10

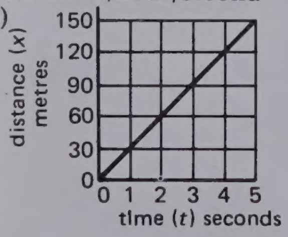


2.11

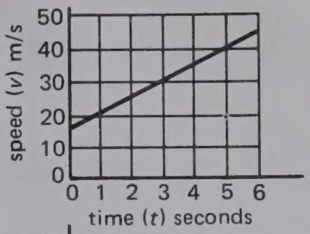


2.12 36.2 miles/hour/second

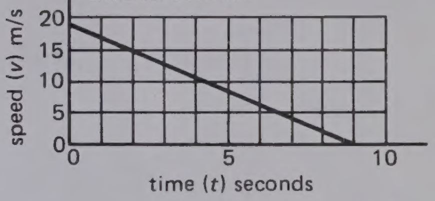
2.13 a)



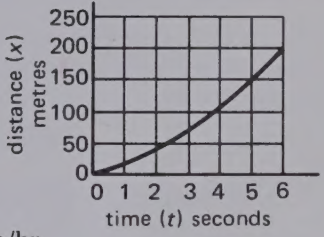
b)



c)



d)



2.14 1.998 km/hr

2.15 a) 1.67 m/s
b) 3.0 m/s

2.16 a) 1 — 4.5s, 1.17 m/s
b) 6 — 8s, 15 m/s
c) 0.7 m/s
d) 0.85 m/s (approx.)
e) 0.4 m (approx.)

2.17 15 m/s, 22.5 m

2.19

speed interval	average a	distance
0 — 10 m/s	3.33 m/s^2	15 m
0 — 20 m/s	3.17	63
0 — 30 m/s	2.52	179
0 — 40 m/s	1.73	462

2.20 50 m/s, 50 m/s

2.21 a) a = 4 m/s^2, d = 42 m/s
b) d = 400 m, v_f = 60 m/s
c) a = 2 m/s^2, v_f = 70 m/s
d) t = 3s, a = 10 m/s^2
e) d = 455 m, t = 7s

2.22 b) 46.6 km/hr
c) Too high
d) Yes, discussion
f) Yes—use outside observer and stop watch

2.24 25.6 m

2.25 Speed of light in air ≃ 3×10^8 m/s

2.27 8.2×10^4 m/s^2

2.28 100 s

2.30 In Chapter 1 we discussed the Greek view of the real world. In Chapter 2 we consider only very simple and idealized forms of motion.

Chapter 3

3.3 0.25 cm/ml^2

3.4 a) 4.3 welfs/surg^2

3.5 a) 80 m
b) 40 m/s

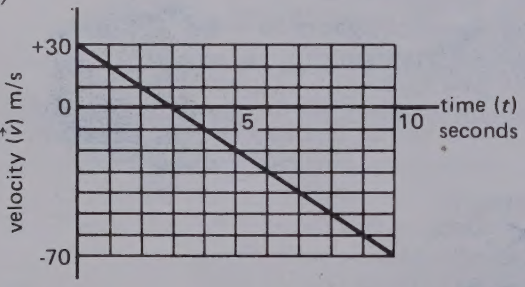
c) 20 m/s
d) 2.0 seconds after release
3.6 $\sqrt{8}$ seconds
3.7 1.4 m
3.8 a) 5 m/s
b) 2.5 m
c) 7.5 m
3.9 45 m
3.10 a) 3.0 s
b) 15 m/s
c) at 1.5 seconds after release
d) 11.25 m
3.11 0.8 m/s
3.16 Yes. 16 m from top of hill.

Chapter 4

4.1 Time is in scalar quantity
4.2 Add a compass to the speedometer
4.3 $\vec{d} = 30$ m West
$V_{av} = 7.8$ m/s
$\vec{V}_{av} = 3.3$ m/s W
4.4

Position	Sign given to the value of		
	d	v	a
A	0	+	—
B	+		—
C	+	0	—
D	+	—	—
E	—	—	—

4.6 a) true
b) true
c) possible
d) false
e) true
4.7 a) 20 m/s upward
b) 20 m/s downward
c) after 4.0 seconds
d) depends what it lands on
e) depends upon the height of the catcher. If caught at height that it was hit from velocity would be 40 m/s downward.
4.8 90 metres
4.9 a) 2 m/s^2 down the ramp
b) 2 m/s
c) 2 m/s along ramp
d) 4 m
e) 2 m/s down the ramp
f) 4 s
4.10 a)



b) 95 m
c) 3.0 + 4.4 s = 7.4 s
4.11 a) 1.0 km
b) 4.0 km/hr
c) 0.7 km NE
d) 2.8 km/hr NE
e) 1.1 km 20° N of E
f) 3km/hr SE
g) 2.3 km
h) 9.2 km/hr
i) 1.3 km 5° S of E
k) 0

4.12 a) 50 s
b) 17 s (16.7 s)
c) 45 s (44.8 s)
4.13 2.06×10^2 km/hr 14° W of North
4.14 12m
4.15 Should aim upstream making an angle of 42° with shoreline.
4.16 94m
4.17 15m 45° South-West and 19° below horizontal line

Chapter 5

5.1 a) 9 units north
b) same
5.2 a) zero
b) No
c) 20 units 30° W of N
5.3 14.2 units 13° N of W
5.9 1.0 N
5.10 a) 25N
b) 5 m/s^2
c) 55 N
5.11 2 kg
5.12 45,000 N
5.19 0.1 kg
5.22 a) 0.2 m/s^2
5.23 a) 20 N up
b) 20 N up
c) 16 N up
d) 24 N up
5.25 i) a) 862 N up
b) 750 N up
c) 638 N up
5.26 a) $\frac{3}{4}$ m/s^2 right
b) $\frac{9}{2}$ N right
c) $\frac{15}{2}$ N right
5.27 6.7 m/s^2
5.28 a) $a_{avg} = 200$ m/s^2, $v_{max} = 780$ m/s
c) $a_{avg} = 240$ m/s^2

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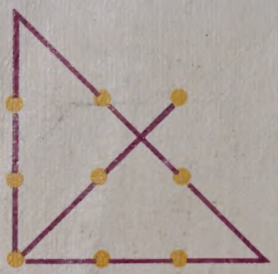

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